UTMB 2022

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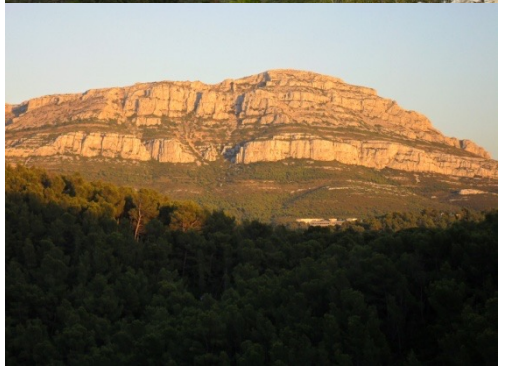
Bruit hors d'équilibre généré par un Gradient de température

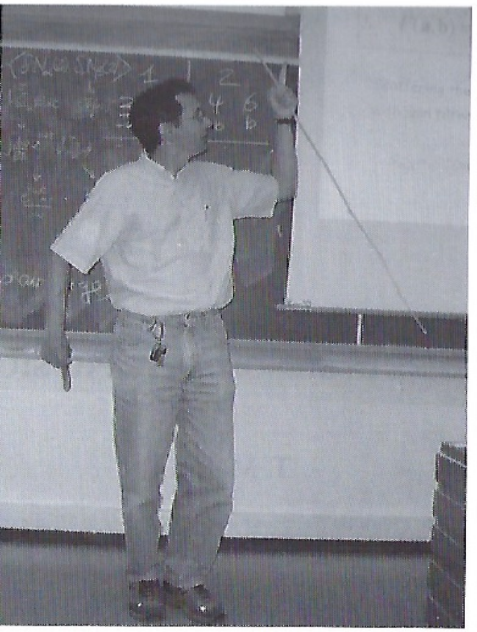


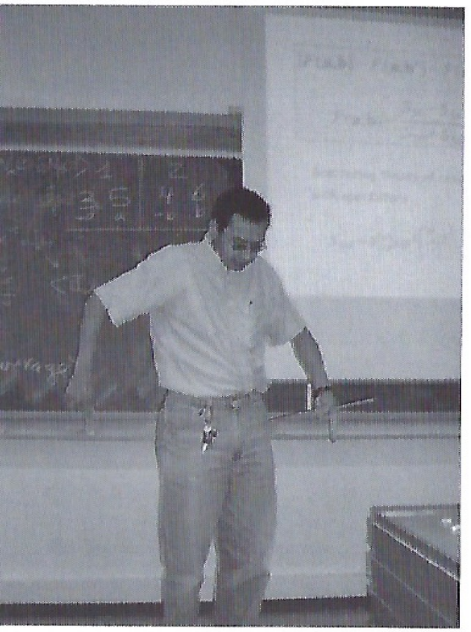
Rencontres de Moriond 1996 (Les Arcs)
T. Martin, G. Montambaux and J. Tran Thanh Van (eds.)
Correlated Fermions and Transport in Mesoscopic Systems
Proceedings of the XXXIst Rencontres de Moriond, Editions Frontières, 1997.

Rencontres du Vietnam 1999 (Hanoi)

Rencontres de Moriond 2001 (Les Arcs)
T. Martin, G. Montambaux and J. Tran Than Van (eds.)
Electronic correlations: from meso to nanophysics
Proceedings of the XXXVI Rencontres de Moriond, EDP Sciences, 2001.

Rencontres du vietnam 2006 (QuyNhon)
T. Martin, D. Mailly, Nguyen Van Hieu, B. Plaçais, and J. Tranh Thanh Van (eds.)
Nanophysics, from fundamentals to applications
Proceedings of the 6th Rencontres du Vietnam, 2006.





« Noise in Mesoscopic Physics »
T. Martin, in Nanophysics: Coherence and Transport, Les Houches, Session LXXXI, edited by H. Bouchiat, Y. Gefen, S. Géron, G. Montambaux, and J. Dalibard (Elsevier, 2005) p. 283.

Contents

22 Rio BARBAIRA ✦✦✦✦

Descente : 3 h

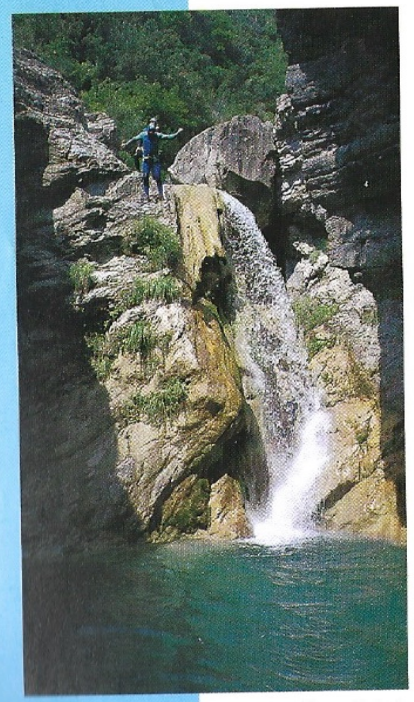
La descente peut commencer par un joli saut de 8 m, depuis le pont, puis une courte marche amène à un toboggan ; puis un rappel évitable rive droite, et la suite n'est qu'un enchaînement de jolies vasques, sauts à vérifier au préalable, mais jamais obligatoires ; la descente est ponctuée de quelques rappels, dans une partie semi-souterraine, dont la voûte est tapissée de concrétions de tufs. On sort de la partie resserrée et une longue partie de marche mène à l'ultime saut, depuis le barrage.

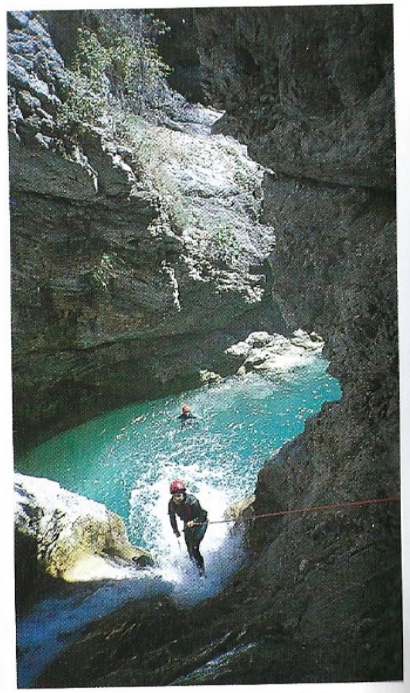
Retour pédestre : 10 mn

Après la dernière vasque, sortir sur la droite et un sentier mène au village.

Pratique

Au restau, repas pantagruélique et accueil chaleureux chez "Roberto" à droite sur la place, vaste terrasse.





Le "must" de la Ligurie, l'aquatique canyon du Barbaira

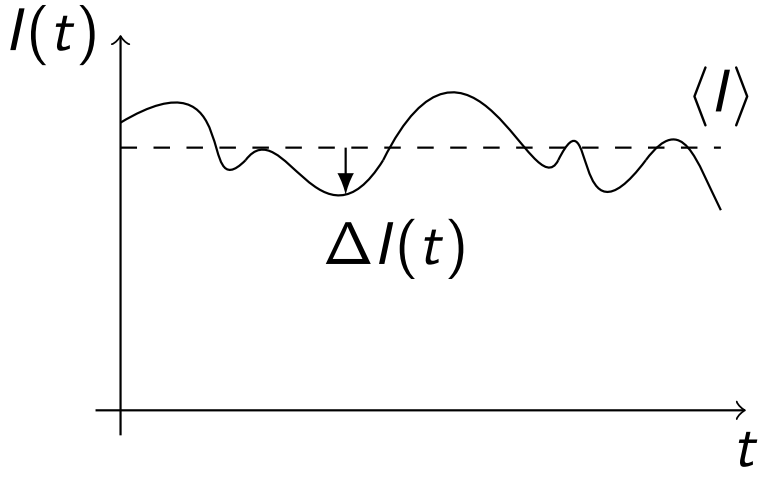
Première décharge d'adrénaline, le saut de 8 m, depuis le pont génois du Barbaira

So much noise...

- Definition

  Noise: *n.* **1.** A sound that is loud, unpleasant, unexpected or undesired
  2. *Physics* A disturbance, especially a random and persistent one, that obscures or reduces the clarity of the signal

- Noise in mesoscopic physics



- fluctuations of the current around its average value
- typically defined as

$$S_{ij} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} dt \int_{-\infty}^{\infty} dt' \times \langle \Delta I_i(t) \Delta I_j(t+t') \rangle$$

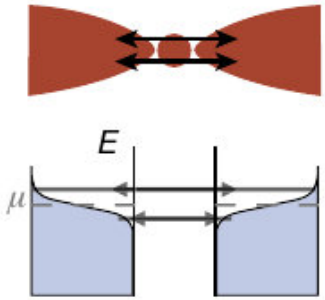
- Just a disturbance?
  - noise contains information not present in the time-averaged current sensitive to the effective charge, statistics of the tunneling carriers
  - a key tool to study quantum transport in nanoscale devices

    ➡ "The noise is the signal" R. Landauer

Different kinds of noise

- Typically viewed as 2 contributions with different underlying physics
- Thermal noise [Johnson, Nature 119, 50 ('27) - Nyquist, Phys. Rev. 32, 110 ('28)]

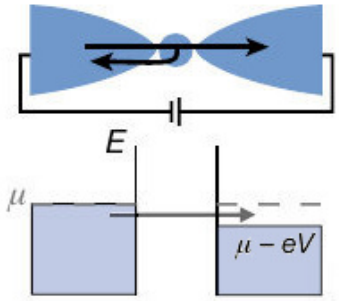
  - first measured by Jonhson, and explained by Nyquist
  - random motion of electrons from thermal fluctuations
  - increases with temperature
  - present even at equilibrium (no bias)



Thermal noise
$T > 0$

- Shot noise [Schottky, Annalen der Physik 362, 541 ('18)]

  - potential bias ⇒ flow of discrete charges
    electrons are transferred or reflected
  - related to the granularity of charge carriers
  - out-of-equilibrium regime

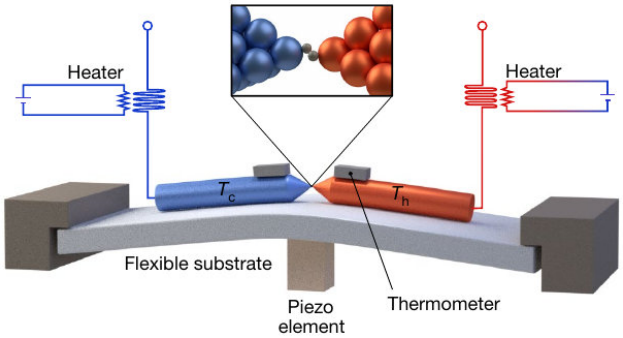


Shot noise
$V \neq 0$

Noise from a temperature difference: Delta-T noise

What if the non-equilibrium situation arises from a temperature bias?

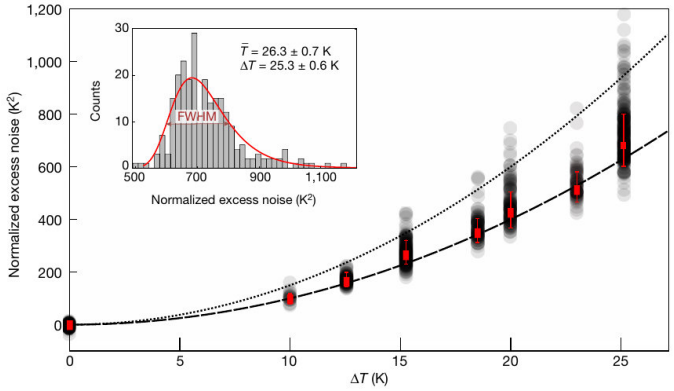
- Noise in T-biased junctions [Shein-Lumbroso et al., Nature 562, 240 ('18)]

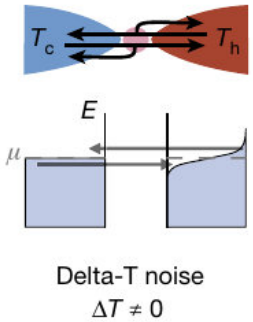


- atomic-scale junction between gold leads
- no voltage but finite temperature bias ΔT
- finite excess noise $\Delta S = S - S_{\mathrm{Th}}$

Completely overlooked contribution to the noise dubbed "ΔT-noise"

- Distinct from the "canonical" contributions



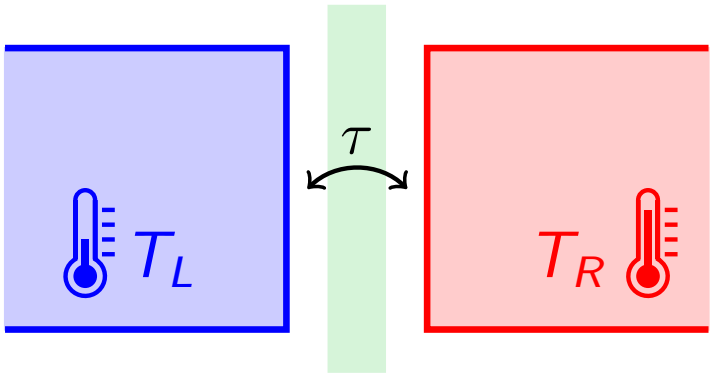


- purely thermal in origin
- relies on partitioning
- out-of-equilibrium only

➡ Delta-T noise corresponds to temperature-activated shot noise

Delta-T noise and scattering theory

- Setup



- normal leads, spinless electrons
- transmission $\tau(E) \simeq \tau$
- no voltage but finite temperature bias $T_{R,L} = \bar{T} \pm \frac{\Delta T}{2}$

- Current $\langle \hat{I}(t) \rangle = \dfrac{e}{h} \displaystyle\int dE\ \tau \left[f_R(E) - f_L(E) \right] = 0,\ \forall (T_R, T_L)$

- Noise $S = 2 \displaystyle\int dt \left[\langle \hat{I}(t) \hat{I}(0) \rangle - \langle \hat{I}(t) \rangle \langle \hat{I}(0) \rangle \right]$

$$= 2 \frac{e^2}{h} \int dE \left\{ \tau \left[f_R \left(1 - f_R\right) + f_L \left(1 - f_L\right) \right] + \tau \left(1 - \tau\right) \left(f_R - f_L\right)^2 \right\}$$

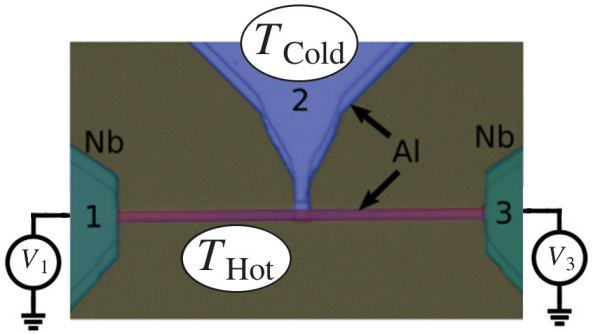
$$\approx 4 \frac{e^2}{h} \tau \bar{T} + \boxed{4 \frac{e^2}{h} \tau \left(1 - \tau\right) \bar{T} \frac{\pi^2 - 6}{9} \left(\frac{\Delta T}{2 \bar{T}} \right)^2}$$

ΔT noise

➡ Delta-T noise is naturally described by scattering theory!

A new probe gaining momentum

- Cold reservoir [Laroque et al., Phys. Rev. Lett. 125, 106801 ('20)]



- tunnel junction under large temperature bias
  $T_{\mathrm{Hot}} \gg T_{\mathrm{Cold}} \sim 0 \Longrightarrow \bar{T} = \Delta T$
- noise thermometry $S = 2\frac{k_B T_{\mathrm{noise}}}{R}$

A new probe gaining momentum

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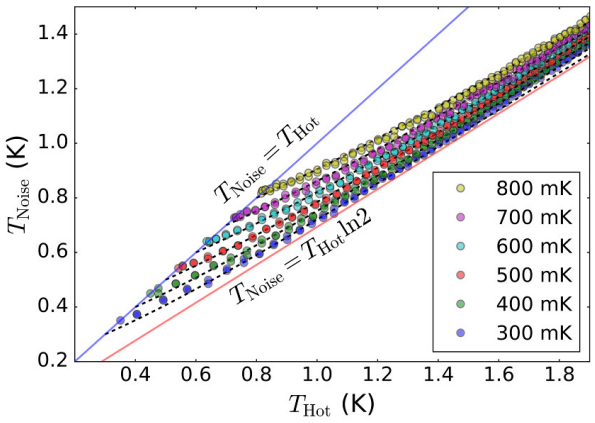
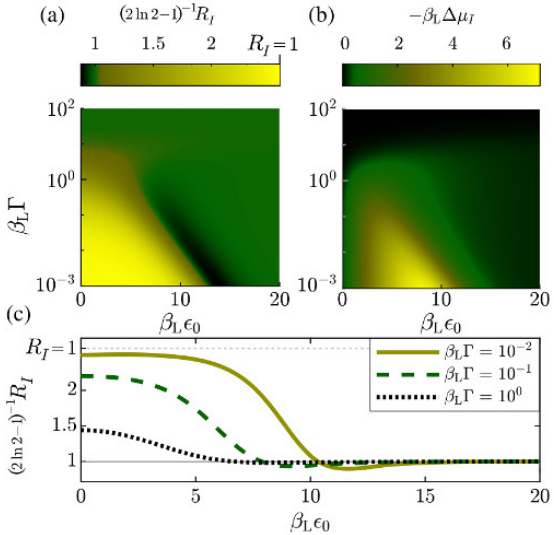


- tunnel junction under large temperature bias $T_{\rm Hot} \gg T_{\rm Cold} \sim 0 \Longrightarrow \bar{T} = \Delta T$
- noise thermometry $S = 2\frac{k_B T_{\rm noise}}{R}$
- noise goes from $\frac{2k_B}{R}\frac{T_{\rm Hot}+T_{\rm Cold}}{2}$ to $\frac{2\log 2 k_B}{R} T_{\rm Hot}$
- extremely good agreement with scattering theory

- Zero current shot noise [Eriksson et al., Phys. Rev. Lett. 127, 136801 ('21)]

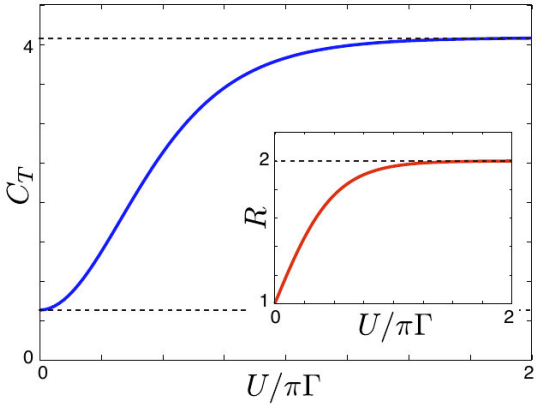


- combined effect of ΔT and $\Delta\mu$ ensuring zero current
- accounts for energy-dependent transmission $\tau(\epsilon)$
- ratio of noise contributions is bounded $R_I = \frac{S_{\rm shot}}{S_{\rm thermal}} \leq 1$
- extension to heat current/noise

- Problem: non-interacting fermions only. Can this be extended?

Effect of interactions on the Delta-T noise

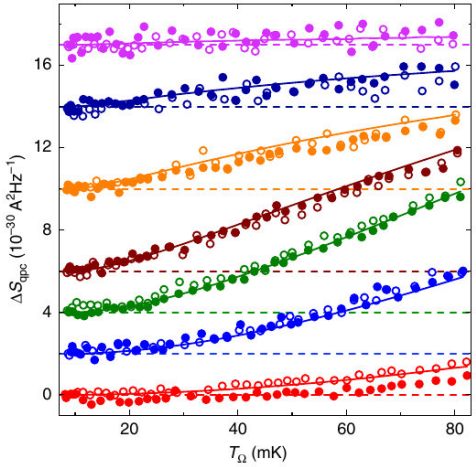
- Deep Kondo regime [Hasegawa et al., arXiv:2008.08839]



- resonant level and local Coulomb repulsion
- extreme regime: $T_L \gg T_R \sim 0 \Longrightarrow \bar{T} = \Delta T$
- leading order contribution with coefficient

$$C_T = 6\zeta(3) - 4\zeta(2) + \left[\tfrac{11}{2}\zeta(3) + (3\log 2 - 4)\,\zeta(2)\right](R-1)^2$$

- Coulomb blockade [Sivré et al., Nat. Comm. 10, 5638 ('19)]



- metallic node connected to a resistance $R = R_K/N$ and one electronic channel
- ΔT-noise measurements reproduced by theory with a Coulomb renormalized transmission
- ΔT-noise and Coulomb interaction lead to an additional quantum heat transport mechanism

Effect of interactions on the Delta-T noise

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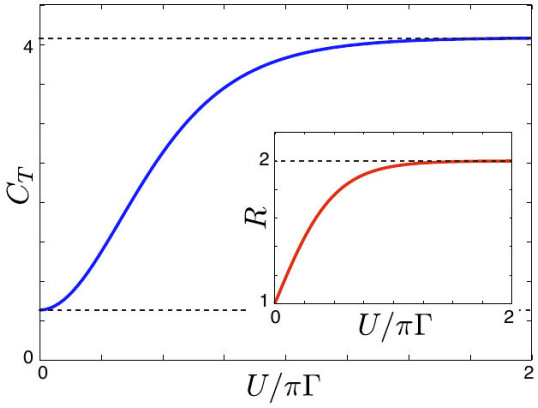

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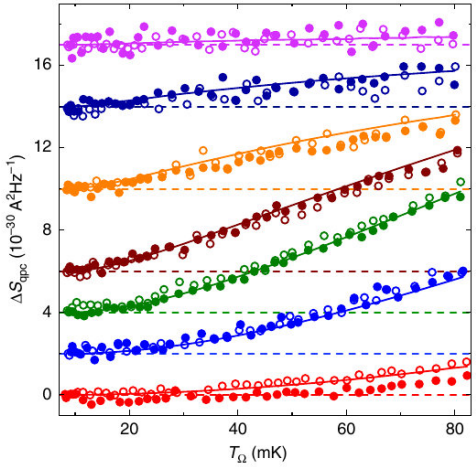

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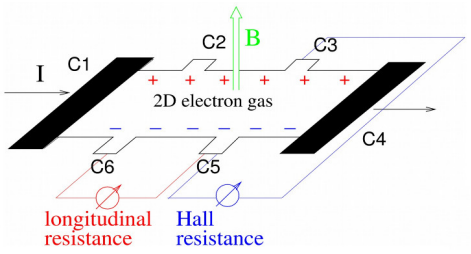
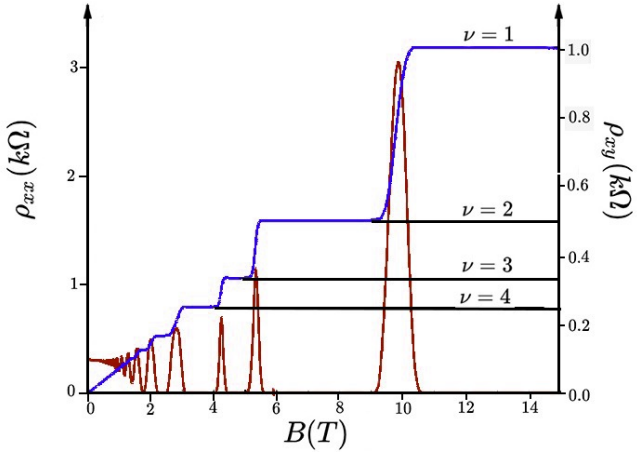
ΔT-noise is a valuable tool to better understand quantum transport phenomena. Sensitivity to interactions worth exploiting in other platforms

Quantum Hall effect

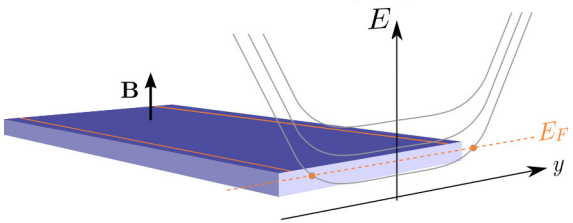
- 2DEG under strong magnetic field (and low temperature)



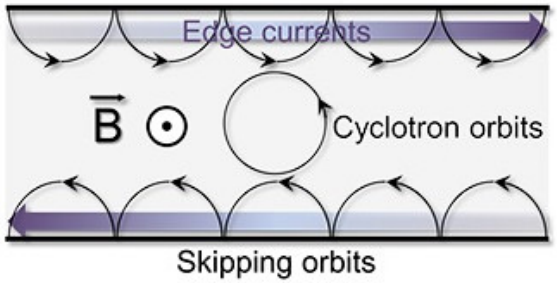
- Resistivity vs. magnetic field
  - sharp peaks in longitudinal resistivity, zero in between
  - plateaus of transverse resistivity at quantized values $\propto 1/n$



- Single electron picture works great
  ➡ Landau levels

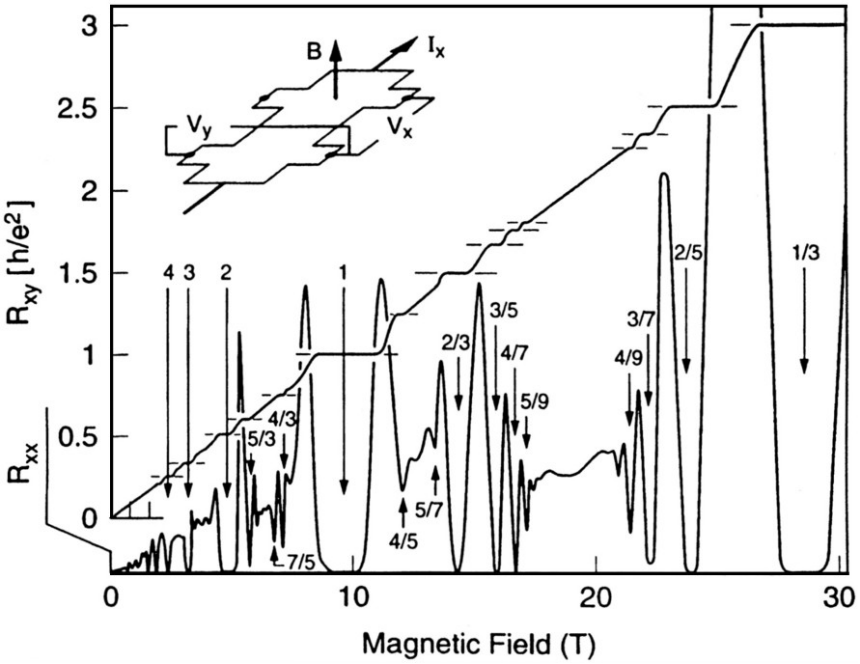


- Edge states
  - classical picture : skipping orbits
  - 1d, chiral edge states
  - topological nature, immune to disorder or defects



Fractional quantum Hall effect

- Better samples ➡ new features in the Hall resistivity

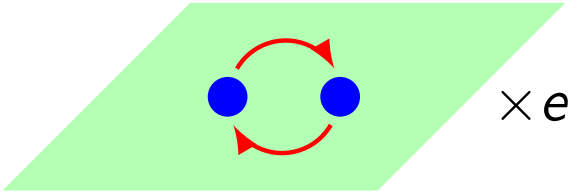


- intermediate plateaus for non-integer values of filling factor
- partially filled Landau levels
- failure of the single electron picture: Coulomb interaction is critical
- Laughlin series $\nu = \frac{1}{2n+1}$

- Similar edge states to IQH...

  ➡ 1D, chiral, topological edge states act as a beam of charge carriers

- ... with a maior difference: building blocks are anyons



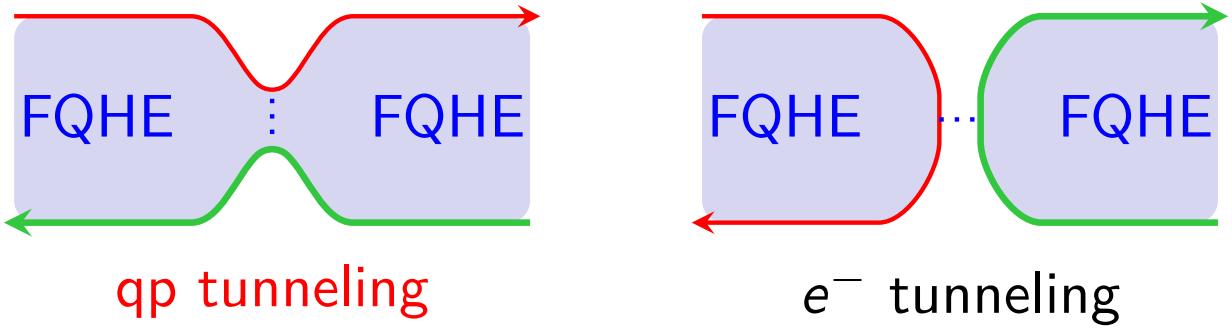
$\times e^{i\phi}$

$e^* = \nu e$

- fractional charge $e^* = \nu e$
- fractional statistics $\phi = \pi\nu$

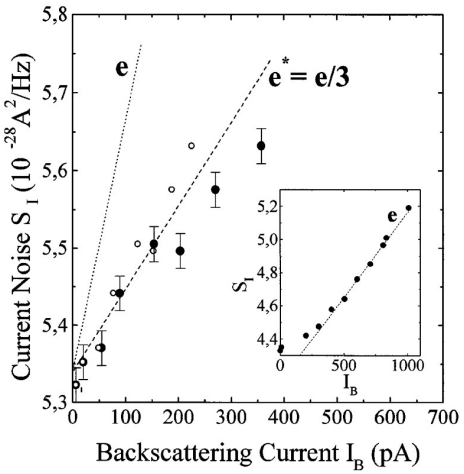
FQH and shot noise

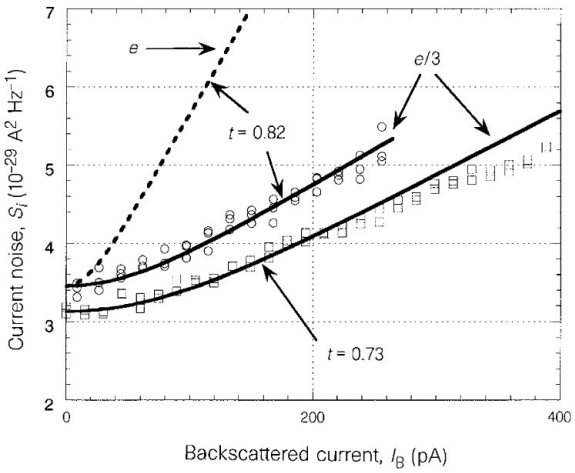
- Quantum point contact: controllable overlap between edge states
  - equivalent to a beam splitter
  - weak vs. strong backscattering



- Experimental proof of the fractional charge at $\nu = 1/3$ [Saminadayar et al., Phys. Rev. Lett. 79, 2526 ('97) - de Picciotto et al., Nature 389, 162 ('97)]

- weak backscatt. $T \ll 1$
- current $I_B = T I_0$
- Poissonian regime $\langle \Delta N^2 \rangle = \langle N \rangle$
- noise $S_B = 2e^* I_B$



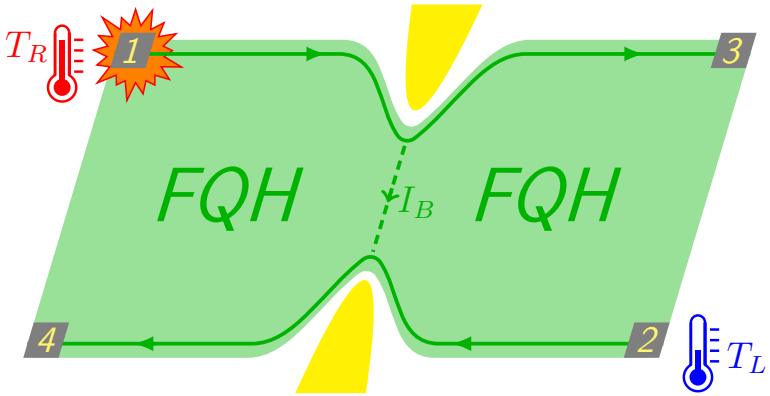


shot noise ➡ granularity of the charge ➡ effective charge $e^* = \frac{e}{3}$

What about ΔT noise? Anything to be learned from it?

(Sketchy) theoretical description

- Setup considered



- Hall bar equipped with a QPC
- Laughlin series $\nu = \frac{1}{2n+1}$
- different temperatures T_R, T_L
- possible potential bias V

- Effective theory: hydrodynamical approach [Wen, IJMP. B 6, 1711 ('92)]
  - incompressible, irrotational liquid ➡ surface waves
  - after quantization, theory formulated in terms of a density operator ρ bosonized effective theory corresponds to a chiral Luttinger liquid model
  - Propagation Hamiltonian $$H_0 = \frac{v}{4\pi}\int dx \left[(\partial_x\phi_R)^2 + (\partial_x\phi_L)^2\right]$$

    with $\rho_{R/L}(x) = \pm e\frac{\sqrt{\nu}}{2\pi}\partial_x\phi_{R/L}(x)$ and $\left[\phi_{R/L}(x), \phi_{R/L}(y)\right] = \pm i\pi \mathrm{Sgn}(x-y)$
  - bosonization identity

$$\Psi_{R/L}(x,t) \propto \frac{1}{2\pi a} e^{-i\frac{1}{\sqrt{\nu}}\phi_{R/L}(x,t)}$$

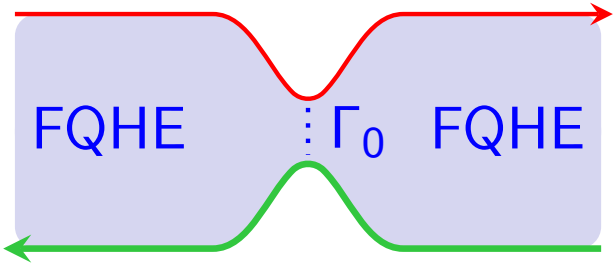
for electrons

$$\psi_{R/L}(x,t) \propto \frac{1}{2\pi a} e^{-i\sqrt{\nu}\phi_{R/L}(x,t)}$$

for quasiparticles

Computing current and noise

- Tunneling Hamiltonian



weak backscattering regime

$$H_{\mathrm{WB}} = \Gamma_0 e^{ie^* Vt} \psi_R^\dagger(0)\psi_L(0) + \mathrm{H.c.}$$

- Backscattered current $\quad I_B(t) = ie^*\Gamma_0 e^{ie^* Vt} \psi_R^\dagger(0,t)\psi_L(0,t) + \mathrm{H.c.}$
  - average value

$$\langle I_B(t)\rangle = 2i\frac{e^*\Gamma_0^2}{(2\pi a)^2}\int_{-\infty}^{\infty} d\tau \sin\left(e^* V\tau\right)\exp\left[\nu\ \mathcal{G}_R\left(\tau\right) + \nu\ \mathcal{G}_L\left(\tau\right)\right]$$

  - Zero-voltage case $\quad \langle I_B(t)\rangle \xrightarrow[V\to 0]{} 0 \quad$ independently of T_R, T_L

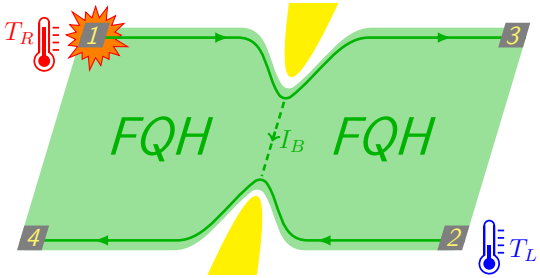
- Fluctuations (leading order in Γ_0)

$$\mathcal{S}_B = 2\int d\tau\ \left[\langle I_B(\tau)I_B(0)\rangle - \langle I_B(\tau)\rangle\langle I_B(0)\rangle\right]$$

$$= \left(\frac{e^*\Gamma_0}{\pi a}\right)^2 \int_{-\infty}^{\infty} d\tau \cos\left(e^* V\tau\right)\exp\left[\nu\mathcal{G}_R\left(\tau\right) + \nu\mathcal{G}_L\left(\tau\right)\right]$$

Main results: weak backscattering regime and $\Delta T \ll \bar{T}$

- Weak temperature difference $T_R \simeq T_L$



- average temperature $\bar{T} = \frac{T_R + T_L}{2}$
- temperature difference $\Delta T = T_R - T_L \ll \bar{T}$

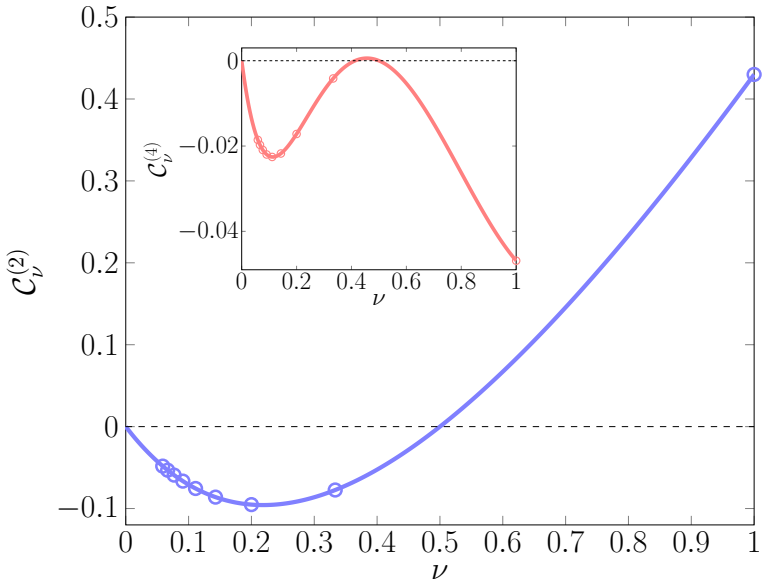
- No simple analytic form $\Longrightarrow$ Perturbative expansion in ΔT

$$\mathcal{S}_B = \mathcal{S}^0_{\mathrm{WB}} \left[1 + \mathcal{C}^{(2)}_\nu \left(\frac{\Delta T}{2\bar{T}} \right)^2 + \mathcal{C}^{(4)}_\nu \left(\frac{\Delta T}{2\bar{T}} \right)^4 \right]$$

with $\mathcal{C}^{(2)}_\nu = \nu \left\{ \frac{\nu}{2\nu+1} \left[\frac{\pi^2}{2} - \psi'(\nu+1) \right] - 1 \right\}$

➡ $\mathcal{C}^{(2)}_{\nu=1}$ recovers scattering theory

➡ $\mathcal{C}^{(2)}_\nu < 0$ for the Laughlin series!



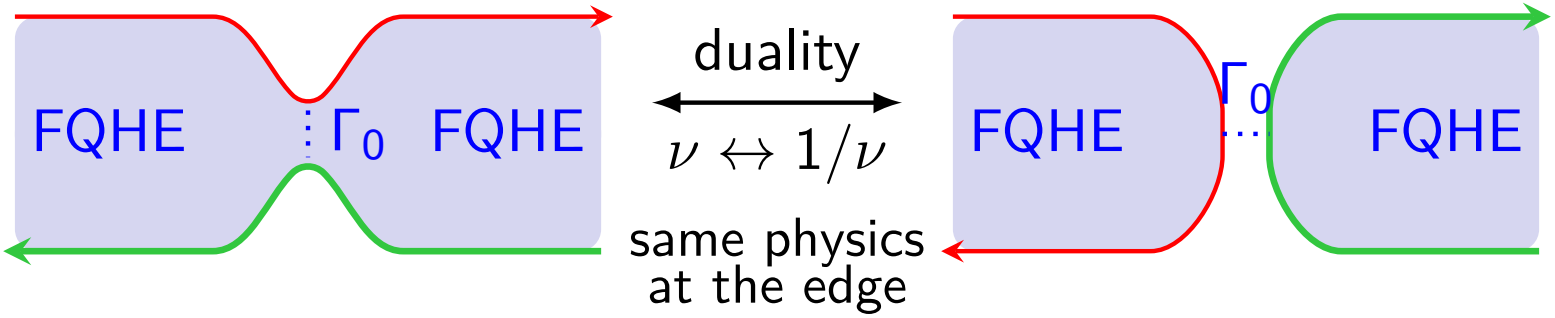
- noise reduction in presence of a bias?

➡ interactions play an important role

Strong backscattering regime

Can this signature be solely attributed to the strong interaction along the edge?

- Strong backscattering regime



$$H_{\mathrm{WB}} = \Gamma_0 \psi_R^\dagger(0)\psi_L(0) + \mathrm{H.c.}$$

$$S_{\Delta T} \propto \mathcal{C}^{(2)} (\Delta T)^2$$

$$\mathcal{C}^{(2)} = \mathcal{C}_\nu^{(2)} < 0$$

qp tunneling

duality

$\nu \leftrightarrow 1/\nu$

$$H_{\mathrm{SB}} = \Gamma \Psi_R^\dagger(0)\Psi_L(0) + \mathrm{H.c.}$$

$$S_{\Delta T} \propto \tilde{\mathcal{C}}^{(2)} (\Delta T)^2$$

$$\tilde{\mathcal{C}}^{(2)} = \mathcal{C}_{1/\nu}^{(2)} > 0$$

e^- tunneling

- Interesting in 2 ways
  - potential experimental detection may be easier
  - importance of quasiparticle tunneling

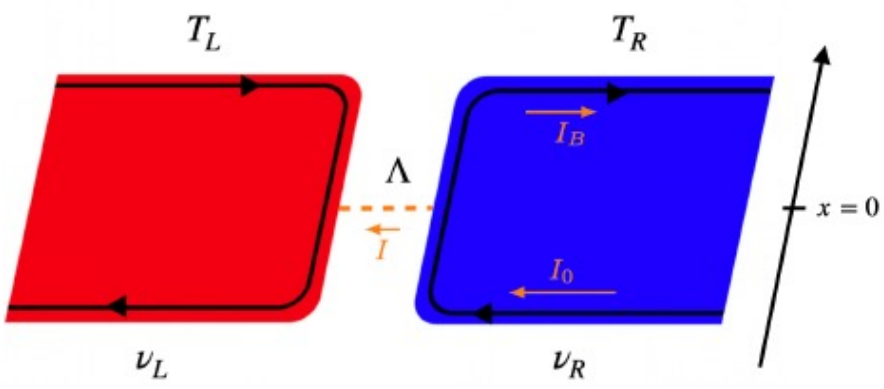
Conclusions

- Peculiar signatures from a temperature bias across a QPC in the FQH
  ➡ Negative ΔT-noise
- Associated with the tunneling of Laughlin quasiparticles
  ➡ suggests an interplay of interaction and statistics
- Reverts to a positive contribution in the strong backscattering regime or under a large voltage bias
- This signature should be accessible experimentally
- Further explore the connection with anyonic statistics
- Extension to more exotic states, beyond Laughlin
  ➡ more complex structure of the edge: neutral modes, $\nu = 5/2$?

Negative delta-T noise in the Fractional Quantum Hall effect
J. Rech, T. Jonckheere, B. Grémaud, and T. Martin,
Phys. Rev. Lett. **125**, 086801 (2020)

Perspectives:

Delta T noise in non symmetric junctions of the FQHE
Exact solution for (1, 1/3) junction via refermionization
Linear contribution on top of quadratic contribution:



Finite frequency delta T noise ?
Microwave photons emitted from a mesoscopic device driven by a temperature gradient

Voltage dependence

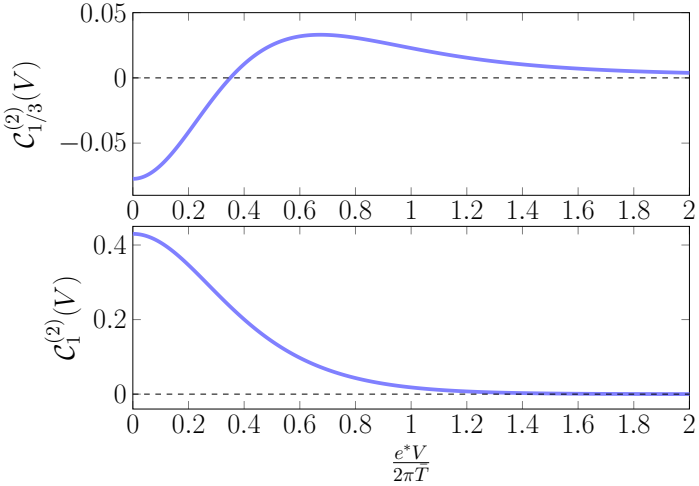
- Finite voltage case ➡ extension to voltage-dependent coefficients

$$\mathcal{S}_B = \mathcal{S}^0_{\mathrm{WB}}\left(V, \bar{T}\right)\left\{1 + \left(\frac{\Delta T}{2\bar{T}}\right)^2 \mathcal{C}^{(2)}_\nu(V) + O\left[\left(\frac{\Delta T}{2\bar{T}}\right)^4\right]\right\}$$

- Analytic expression

$$\mathcal{C}^{(2)}_\nu(V) = -\nu + \frac{\nu^2 + \left(\frac{e^* V}{2\pi \bar{T}}\right)^2}{2\nu + 1}\left\{-2\pi \mathrm{Im}\ \psi\left(\nu + 1 + i\frac{e^* V}{2\pi \bar{T}}\right)\tanh\left(\frac{e^* V}{2\bar{T}}\right)\right.$$
$$\left. + \frac{\pi^2}{2} + 2\left[\mathrm{Im}\ \psi\left(\nu + 1 + i\frac{e^* V}{2\pi \bar{T}}\right)\right]^2 - \mathrm{Re}\ \psi'\left(\nu + 1 + i\frac{e^* V}{2\pi \bar{T}}\right)\right\}$$

- Main results

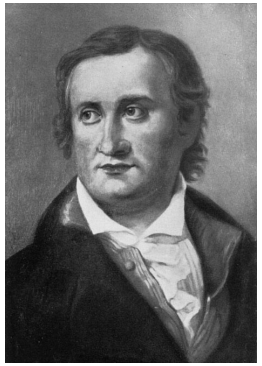


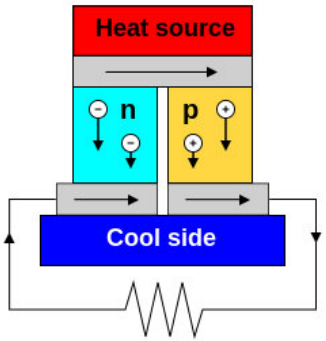
- voltage-dependent coefficients $\mathcal{C}^{(2)}_\nu(V)$ and $\mathcal{C}^{(4)}_\nu(V)$
- ΔT contributions drowned out at large voltage
- $\mathcal{C}^{(2)}_\nu(V)$ sign flips at finite voltage

Should we worry about thermoelectric effects?

Thermoelectric effects vs. Delta-T noise

- Seebeck effect

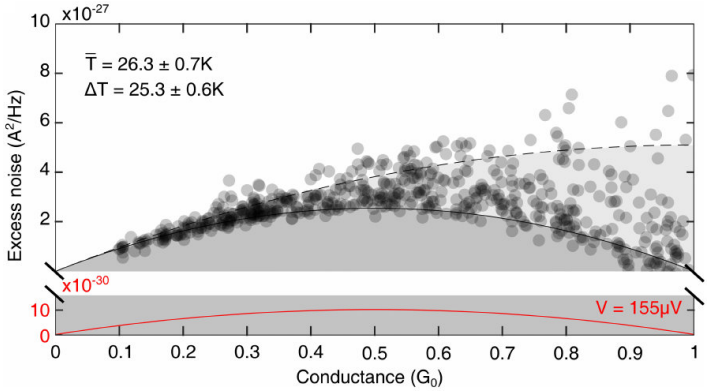




- electric current arising from a temperature difference
- can in turn lead to regular shot noise

Is ΔT noise just the shot noise of this thermally induced current?

- Electrons in a normal junction [Shein-Lumbroso et al., Nature 562, 240 ('18)]



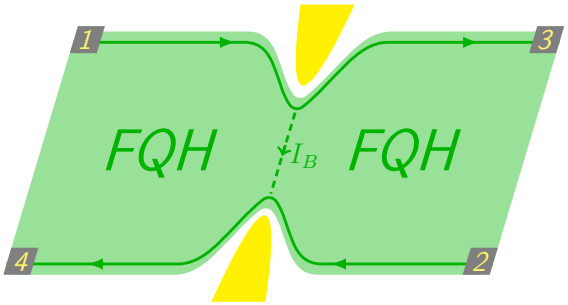
- measure the thermoelectric voltage
- calculate the associated shot noise

➡ orders of magnitude smaller than ΔT-noise

- Why it should matter even less here?
  - incompressible bulk
  - chiral edge states, overlapping only at the QPC
  - no equilibration expected over micron-size distances

Experimental realization I

- Measuring backscattered current fluctuations



- no direct access to the noise $\mathcal{S}_B$
- possible to measure currents I_3, I_4 and their auto- and cross-correlations $\mathcal{S}_{33}, \mathcal{S}_{44}$ and $\mathcal{S}_{34}$

- From backscattered current to measurable currents

$$\langle I_3(x_3,t)\rangle = -\mathcal{I}_B + \frac{\nu e^2}{2\pi} V$$

$$\langle I_4(x_4,t)\rangle = \mathcal{I}_B$$

- From backscattered current fluctuations to cross-correlations
  - Cross-correlations

$$S_{34}(t-t') = \langle I_3(x_3,t) I_4(x_4,t')\rangle - \langle I_3(x_3,t)\rangle \langle I_4(x_4,t')\rangle$$

  - Connection with backscattered current fluctuations

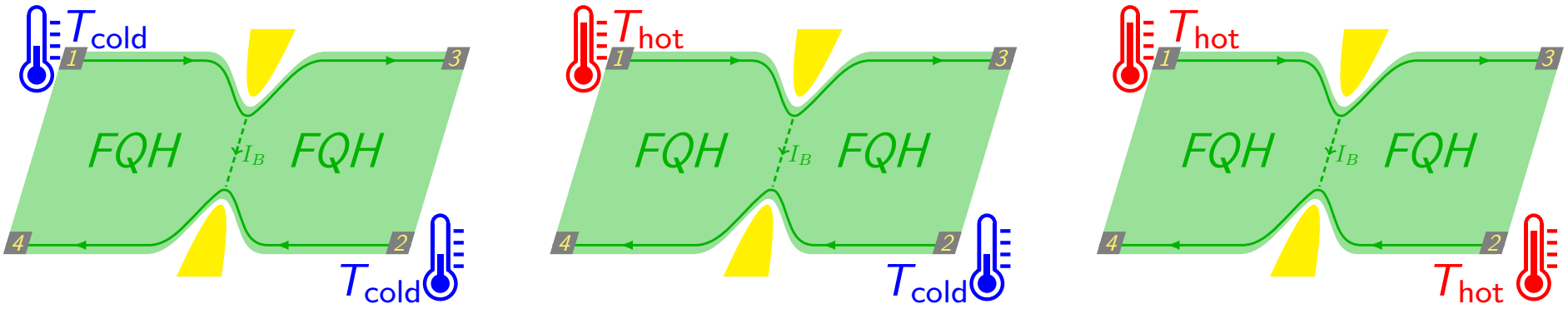
$$\mathcal{S}_{34} = 2\left(T_R + T_L\right)\frac{\partial \mathcal{I}_B}{\partial V} - \mathcal{S}_B$$

➡ $\mathcal{S}_B$ can be measured via $\mathcal{S}_B = 2(T_L + T_R)G_4 - \mathcal{S}_{34}$

Experimental realization II

- Reparametrization $\begin{cases} T_L = \bar{T} - \frac{\Delta T}{2} \rightarrow T_L = T_{\text{cold}} \\ T_R = \bar{T} + \frac{\Delta T}{2} \rightarrow T_R = T_{\text{hot}} = T_{\text{cold}} + \Delta T \end{cases}$

- Three-step measurement



① ➡ $\mathcal{S}\left(T_{\text{cold}}, T_{\text{cold}}\right)$ ② ➡ $\mathcal{S}\left(T_{\text{hot}}, T_{\text{cold}}\right)$ ③ ➡ $\mathcal{S}\left(T_{\text{hot}}, T_{\text{hot}}\right)$

① ➡ measure at base temperature T_{cold}

② ➡ heat up one entry port at T_{hot}

③ ➡ heat up the other entry port at T_{hot}

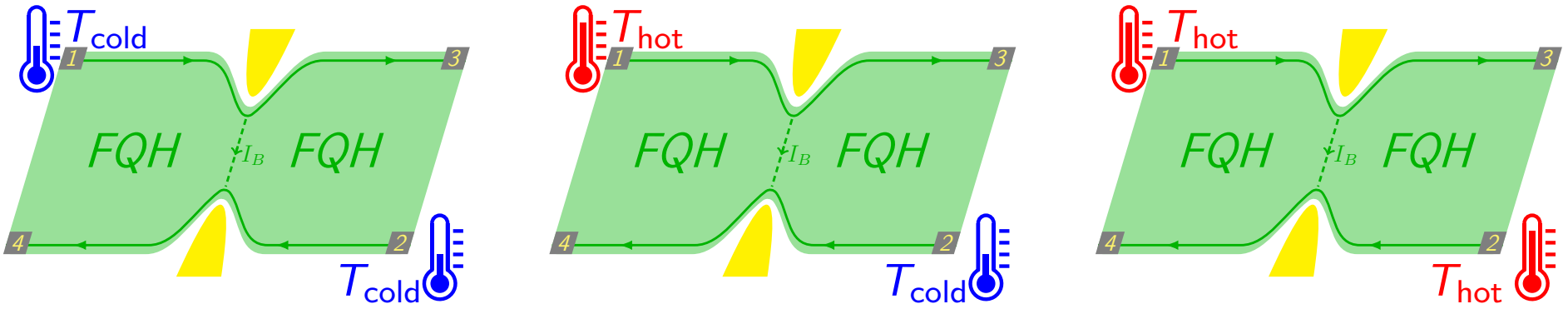
$$\begin{aligned} \Delta\mathcal{S}_{II} &= \mathcal{S}_{②} - \mathcal{S}_{①} \\ &= \mathcal{S}_{\text{cold}} \left(2\nu - 1\right) \frac{\Delta T}{2T_{\text{cold}}} \\ &\quad \times \left[1 + \left(\nu - 1 + \frac{\mathcal{C}_\nu^{(2)}}{2\nu - 1}\right) \frac{\Delta T}{2T_{\text{cold}}}\right] \end{aligned}$$

$$\begin{aligned} \Delta\mathcal{S}_{III} &= \mathcal{S}_{②} - \frac{1}{2}\left(\mathcal{S}_{①} + \mathcal{S}_{③}\right) \\ &= \mathcal{S}_{\text{cold}} \left[\mathcal{C}_\nu^{(2)} - \left(2\nu - 1\right)\left(\nu - 1\right)\right] \left(\frac{\Delta T}{2T_{\text{cold}}}\right)^2 \end{aligned}$$

Experimental realization II

- Reparametrization $\begin{cases} T_L = \bar{T} - \frac{\Delta T}{2} \rightarrow T_L = T_{\text{cold}} \\ T_R = \bar{T} + \frac{\Delta T}{2} \rightarrow T_R = T_{\text{hot}} = T_{\text{cold}} + \Delta T \end{cases}$

- Three-step measurement



① ➡ $\mathcal{S}\left(T_{\text{cold}}, T_{\text{cold}}\right)$ ② ➡ $\mathcal{S}\left(T_{\text{hot}}, T_{\text{cold}}\right)$ ③ ➡ $\mathcal{S}\left(T_{\text{hot}}, T_{\text{hot}}\right)$

① ➡ measure at base temperature T_{cold}

② ➡ heat up one entry port at T_{hot}

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$$\begin{aligned} \Delta\mathcal{S}_{II} &= \mathcal{S}_{②} - \mathcal{S}_{①} \\ &= \mathcal{S}_{\text{cold}} \left(2\nu - 1\right) \frac{\Delta T}{2T_{\text{cold}}} \\ &\quad \times \left[1 + \left(\nu - 1 + \frac{\mathcal{C}_\nu^{(2)}}{2\nu - 1}\right) \frac{\Delta T}{2T_{\text{cold}}}\right] \end{aligned}$$

$$\begin{aligned} \Delta\mathcal{S}_{III} &= \mathcal{S}_{②} - \tfrac{1}{2}\left(\mathcal{S}_{①} + \mathcal{S}_{③}\right) \\ &= \mathcal{S}_{\text{cold}} \underbrace{\left[\mathcal{C}_\nu^{(2)} - \left(2\nu - 1\right)\left(\nu - 1\right)\right]}_{< 0 \text{ for } \nu = \frac{1}{2n+1}} \left(\frac{\Delta T}{2T_{\text{cold}}}\right)^2 \end{aligned}$$