Bruit hors d'équilibre généré par un Gradient de température
Rencontres de Moriond 1996 (Les Arcs)
T. Martin, G. Montambaux and J. Tran Thanh Van (eds.)
**Correlated Fermions and Transport in Mesoscopic Systems**

Rencontres du Vietnam 1999 (Hanoi)

Rencontres de Moriond 2001 (Les Arcs)
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**Electronic correlations: from meso to nanophysics**

Rencontres du Vietnam 2006 (QuyNhon)
T. Martin, D. Mailly, Nguyen Van Hieu, B. Plaçais, and J. Tranh Thanh Van (eds.)
**Nanophysics, from fundamentals to applications**
« Noise in Mesoscopic Physics »

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Rio BARBAIRA

Descente : 3 h
La descente peut commencer par un joli saut de 8 m, depuis le pont, puis une courte marche amène à un toboggan ; puis un rappel évitable rive droite, et la suite n'est qu'un enchaînement de jolies vasques, sauts à vérifier au préalable, mais jamais obligatoires ; la descente est ponctuée de quelques rappels, dans une partie semi-souterraine, dont la voûte est tapissée de concrétions de tufs. On sort de la partie resserrée et une longue partie de marche mène à l'ultime saut, depuis le barrage.

Retour pédestre : 10 mn
Après la dernière vasque, sortir sur la droite et un sentier mène au village.

Pratique
Au retour, repas pionigréal et accueil chaleureux chez "Roberto" à droite sur la place, vaste terrasse.
So much noise...

- **Definition**
  
  **Noise**: *n.* 1. A sound that is loud, unpleasant, unexpected or undesired
  
  2. **Physics** A disturbance, especially a random and persistent one, that obscures or reduces the clarity of the signal

- **Noise in mesoscopic physics**
  
  - **fluctuations** of the current around its average value
  
  - typically defined as
    
    \[
    S_{ij} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt \int_{-\infty}^{\infty} dt' \langle \Delta l_i(t) \Delta l_j(t + t') \rangle
    \]

- **Just a disturbance?**
  
  - noise contains information not present in the time-averaged current sensitive to the effective charge, statistics of the tunneling carriers
  
  - a **key tool** to study quantum transport in nanoscale devices
    
    ➡️ "The noise is the signal" R. Landauer
Different kinds of noise

- Typically viewed as 2 contributions with different underlying physics

- **Thermal noise** [Johnson, Nature 119, 50 ('27) - Nyquist, Phys. Rev. 32, 110 ('28)]
  - first measured by Johnson, and explained by Nyquist
  - random motion of electrons from thermal fluctuations
  - increases with temperature
  - present *even at equilibrium* (no bias)

- **Shot noise** [Schottky, Annalen der Physik 362, 541 ('18)]
  - potential bias $\Rightarrow$ flow of discrete charges
  - electrons are transferred or reflected
  - related to the granularity of charge carriers
  - out-of-equilibrium regime
Noise from a temperature difference: Delta-\(T\) noise

What if the non-equilibrium situation arises from a \textit{temperature bias}?

- Noise in \(T\)-biased junctions [Shein-Lumbroso et al., Nature 562, 240 (’18)]
  - atomic-scale junction between gold leads
  - no voltage but finite temperature bias \(\Delta T\)
  - finite excess noise \(\Delta S = S - S_{\text{Th}}\)

Completely overlooked contribution to the noise dubbed \textit{“\(\Delta T\)-noise”}

- Distinct from the \textit{“canonical”} contributions
  - purely \textit{thermal} in origin
  - relies on \textit{partitioning}
  - out-of-equilibrium only

\(\rightarrow\) Delta-\(T\) noise corresponds to temperature-activated shot noise
Delta-\(T\) noise and scattering theory

**Setup**
- normal leads, spinless electrons
- transmission \(\tau(E) \approx \tau\)
- no voltage but finite temperature bias \(T_{R,L} = \bar{T} \pm \frac{\Delta T}{2}\)

**Current**
\[
\langle \hat{i}(t) \rangle = \frac{e}{\hbar} \int dE \tau [f_R(E) - f_L(E)] = 0, \forall (T_R, T_L)
\]

**Noise**
\[
S = 2 \int dt \left[ \langle \hat{i}(t)\hat{i}(0) \rangle - \langle \hat{i}(t) \rangle \langle \hat{i}(0) \rangle \right]
\]
\[
= 2 \frac{e^2}{\hbar} \int dE \left\{ \tau [f_R (1 - f_R) + f_L (1 - f_L)] + \tau (1 - \tau) (f_R - f_L)^2 \right\}
\]
\[
\approx 4 \frac{e^2}{\hbar} \tau \bar{T} + 4 \frac{e^2}{\hbar} \tau (1 - \tau) \frac{\bar{T}}{9} \left( \frac{\Delta T}{2\bar{T}} \right)^2
\]

\(\Delta T\) noise

**Delta-\(T\) noise is naturally described by scattering theory!**
A new probe gaining momentum

- Cold reservoir [Laroque et al., Phys. Rev. Lett. 125, 106801 ('20)]

- tunnel junction under large temperature bias
  \[ T_{\text{Hot}} \gg T_{\text{Cold}} \sim 0 \implies \bar{T} = \Delta T \]

- noise thermometry \( S = 2 \frac{k_B T_{\text{noise}}}{R} \)
A new probe gaining momentum

- Cold reservoir [Laroque et al., Phys. Rev. Lett. 125, 106801 ('20)]
  
  tunnel junction under large temperature bias
  \[ T_{\text{Hot}} \gg T_{\text{Cold}} \sim 0 \implies \bar{T} = \Delta T \]

- noise thermometry \[ S = 2 \frac{k_B T_{\text{noise}}}{R} \]

- noise goes from \[ \frac{2k_B}{R} \frac{T_{\text{Hot}} + T_{\text{Cold}}}{2} \] to \[ \frac{2 \log 2k_B}{R} T_{\text{Hot}} \]

- extremely good agreement with scattering theory


  combined effect of \( \Delta T \) and \( \Delta \mu \) ensuring zero current

  accounts for energy-dependent transmission \( \tau(\epsilon) \)

  ratio of noise contributions is bounded
  \[ R_I = \frac{S_{\text{shot}}}{S_{\text{thermal}}} \leq 1 \]

  extension to heat current/noise

- Problem: non-interacting fermions only. Can this be extended?
Effect of interactions on the Delta-\(T\) noise

- **Deep Kondo regime** [Hasegawa et al., arXiv:2008.08839]
  - resonant level and local Coulomb repulsion
  - extreme regime: \( T_L \gg T_R \sim 0 \implies \bar{T} = \Delta T \)
  - leading order contribution with coefficient
    \( C_T = 6\zeta(3) - 4\zeta(2) + \left[ \frac{11}{2} \zeta(3) + (3 \log 2 - 4) \zeta(2) \right] (R - 1)^2 \)

- **Coulomb blockade** [Sivré et al., Nat. Comm. 10, 5638 (‘19)]
  - metallic node connected to a resistance
    \( R = R_K/N \) and one electronic channel
  - \( \Delta T \)-noise measurements reproduced by theory with a Coulomb renormalized transmission
  - \( \Delta T \)-noise and Coulomb interaction lead to an additional quantum heat transport mechanism
Effect of interactions on the Delta-\(T\) noise

  - resonant level and local Coulomb repulsion
  - extreme regime: \(T_L \gg T_R \sim 0 \implies \bar{T} = \Delta T\)
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    \[C_T = 6\zeta(3) - 4\zeta(2) + \left[\frac{11}{2}\zeta(3) + (3 \log 2 - 4)\zeta(2)\right] (R - 1)^2\]

- Coulomb blockade [Sivré et al., Nat. Comm. 10, 5638 (‘19)]
  - metallic node connected to a resistance \(R = R_K/N\) and one electronic channel
  - \(\Delta T\)-noise measurements reproduced by theory with a Coulomb renormalized transmission
  - \(\Delta T\)-noise and Coulomb interaction lead to an additional quantum heat transport mechanism

\(\Delta T\)-noise is a valuable tool to better understand quantum transport phenomena. Sensitivity to interactions worth exploiting in other platforms.
Quantum Hall effect

- 2D electron gas (2DEG) under strong magnetic field (and low temperature)
- Resistivity vs. magnetic field
  - sharp peaks in longitudinal resistivity, zero in between
  - plateaus of transverse resistivity at quantized values $\propto \frac{1}{n}$
- Single electron picture works great
  - Landau levels
- Edge states
  - classical picture: skipping orbits
  - 1d, chiral edge states
  - topological nature, immune to disorder or defects
Fractional quantum Hall effect

- Better samples ➞ new features in the Hall resistivity
  - intermediate plateaus for non-integer values of filling factor
  - partially filled Landau levels
  - failure of the single electron picture: Coulomb interaction is critical
  - Laughlin series $\nu = \frac{1}{2n+1}$

- Similar edge states to IQH...
  - 1D, chiral, topological edge states act as a beam of charge carriers
  - ... with a major difference: building blocks are anyons

- $e^* = \nu e$
  - fractional charge $e^* = \nu e$
  - fractional statistics $\phi = \pi \nu$
FQH and shot noise

- Quantum point contact: controllable overlap between edge states
  - equivalent to a beam splitter
  - weak vs. strong backscattering

- Experimental proof of the fractional charge at $\nu = 1/3$ [Saminadayar et al., Phys. Rev. Lett. 79, 2526 (’97) - de Picciotto et al., Nature 389, 162 (’97)]
  - weak backscatt. $T \ll 1$
  - current $I_B = T I_0$
  - Poissonian regime $\langle \Delta N^2 \rangle = \langle N \rangle$
  - noise $S_B = 2 e^* I_B$

shot noise $\rightarrow$ granularity of the charge $\rightarrow$ effective charge $e^* = \frac{e}{3}$

What about $\Delta T$ noise? Anything to be learned from it?
(Sketchy) theoretical description

- Setup considered
  - Hall bar equipped with a QPC
  - Laughlin series $\nu = \frac{1}{2n+1}$
  - different temperatures $T_R, T_L$
  - possible potential bias $V$

- Effective theory: hydrodynamical approach [Wen, IJMP. B 6, 1711 ('92)]
  - incompressible, irrotational liquid $\rightarrow$ surface waves
  - after quantization, theory formulated in terms of a density operator $\rho$
    bosonized effective theory corresponds to a chiral Luttinger liquid model

- Propagation Hamiltonian
  $$H_0 = \frac{\nu}{4\pi} \int dx \left[ (\partial_x \phi_R)^2 + (\partial_x \phi_L)^2 \right]$$
  with $\rho_{R/L}(x) = \pm e^{\frac{\nu}{2\pi}} \partial_x \phi_{R/L}(x)$ and $[\phi_{R/L}(x), \phi_{R/L}(y)] = \pm i\pi \text{Sgn}(x - y)$

- bosonization identity
  $$\Psi_{R/L}(x, t) \propto \frac{1}{2\pi a} e^{-i\frac{1}{\sqrt{\nu}}\phi_{R/L}(x, t)}$$
  for electrons
  $$\psi_{R/L}(x, t) \propto \frac{1}{2\pi a} e^{-i\sqrt{\nu}\phi_{R/L}(x, t)}$$
  for quasiparticles
Computing current and noise

- **Tunneling Hamiltonian**
  \[ H_{WB} = \Gamma_0 e^{ie^* V_t} \psi_R^\dagger(0) \psi_L(0) + \text{H.c.} \]

  - **Backscattered current**
    \[ I_B(t) = ie^* \Gamma_0 e^{ie^* V_t} \psi_R^\dagger(0, t) \psi_L(0, t) + \text{H.c.} \]
    
      - **average value**
      \[ \langle I_B(t) \rangle = 2i \frac{e^* \Gamma_0^2}{(2\pi a)^2} \int_{-\infty}^{\infty} d\tau \sin (e^* V \tau) \exp [\nu G_R(\tau) + \nu G_L(\tau)] \]

    - **Zero-voltage case**
      \[ \langle I_B(t) \rangle \xrightarrow{V \to 0} 0 \] independently of \( T_R, T_L \)

  - **Fluctuations (leading order in \( \Gamma_0 \))**
    \[ S_B = 2 \int d\tau \ [\langle I_B(\tau) I_B(0) \rangle - \langle I_B(\tau) \rangle \langle I_B(0) \rangle] \]
    \[ = \left( \frac{e^* \Gamma_0}{\pi a} \right)^2 \int_{-\infty}^{\infty} d\tau \cos (e^* V \tau) \exp [\nu G_R(\tau) + \nu G_L(\tau)] \]
Main results: weak backscattering regime and $\Delta T \ll \bar{T}$

- Weak temperature difference $T_R \simeq T_L$
  - average temperature $\bar{T} = \frac{T_R + T_L}{2}$
  - temperature difference $\Delta T = T_R - T_L \ll \bar{T}$

- No simple analytic form $\implies$ Perturbative expansion in $\Delta T$

\[ S_B = S_{WB}^0 \left[ 1 + C^{(2)}_\nu \left( \frac{\Delta T}{2\bar{T}} \right)^2 + C^{(4)}_\nu \left( \frac{\Delta T}{2\bar{T}} \right)^4 \right] \]

with $C^{(2)}_\nu = \nu \left\{ \frac{\nu}{2\nu + 1} \left[ \frac{\pi^2}{2} - \psi'(\nu + 1) \right] - 1 \right\}$

$\implies C^{(2)}_{\nu=1}$ recovers scattering theory

$\implies C^{(2)}_\nu < 0$ for the Laughlin series!

- noise reduction in presence of a bias?
  $\implies$ interactions play an important role
Strong backscattering regime

Can this signature be solely attributed to the strong interaction along the edge?

- Strong backscattering regime

\[ H_{WB} = \Gamma_0 \psi_R^\dagger (0) \psi_L (0) + \text{H.c.} \]

\[ S_{\Delta T} \propto C^{(2)} (\Delta T)^2 \]

\[ C^{(2)} = C^{(2)}_\nu < 0 \]

qp tunneling

\[ H_{SB} = \Gamma \psi_R^\dagger (0) \psi_L (0) + \text{H.c.} \]

\[ S_{\Delta T} \propto \tilde{C}^{(2)} (\Delta T)^2 \]

\[ \tilde{C}^{(2)} = C^{(2)}_{1/\nu} > 0 \]

e\text{tunneling}

- Interesting in 2 ways
  - potential experimental detection may be easier
  - importance of quasiparticle tunneling
Conclusions

- Peculiar signatures from a temperature bias across a QPC in the FQH
  - Negative $\Delta T$-noise

- Associated with the tunneling of Laughlin quasiparticles
  - suggests an interplay of interaction and statistics

- Reverts to a positive contribution in the strong backscattering regime
  or under a large voltage bias

- This signature should be accessible experimentally

- Further explore the connection with anyonic statistics

- Extension to more exotic states, beyond Laughlin
  - more complex structure of the edge: neutral modes, $\nu = 5/2$?

**Negative delta-\textit{T} noise in the Fractional Quantum Hall effect**

J. Rech, T. Jonckheere, B. Grémaud, and T. Martin,

Perspectives:

Delta T noise in non-symmetric junctions of the FQHE
Exact solution for (1, 1/3) junction via refermionization
Linear contribution on top of quadratic contribution:

Finite frequency delta T noise?
Microwave photons emitted from a mesoscopic device driven by a temperature gradient
Voltage dependence

- Finite voltage case ➞ extension to voltage-dependent coefficients

\[ S_B = S_{WB}^0 (V, \bar{T}) \left\{ 1 + \left( \frac{\Delta T}{2 \bar{T}} \right)^2 C_\nu^{(2)}(V) + O \left( \frac{\Delta T}{2 \bar{T}} \right)^4 \right\} \]

- Analytic expression

\[ C_\nu^{(2)}(V) = -\nu + \frac{\nu^2 + \left( \frac{e^* V}{2\pi \bar{T}} \right)^2}{2\nu + 1} \left\{ -2\pi \text{Im} \, \psi \left( \nu + 1 + i \frac{e^* V}{2\pi \bar{T}} \right) \tanh \left( \frac{e^* V}{2 \bar{T}} \right) \right. \]
\[ \left. + \frac{\pi^2}{2} + 2 \left[ \text{Im} \, \psi \left( \nu + 1 + i \frac{e^* V}{2\pi \bar{T}} \right) \right]^2 - \text{Re} \, \psi' \left( \nu + 1 + i \frac{e^* V}{2\pi \bar{T}} \right) \right\} \]

- Main results

- voltage-dependent coefficients \( C_\nu^{(2)}(V) \) and \( C_\nu^{(4)}(V) \)
- \( \Delta T \) contributions drowned out at large voltage
- \( C_\nu^{(2)}(V) \) sign flips at finite voltage

Should we worry about thermoelectric effects?
Thermoelectric effects vs. Delta-\(T\) noise

- **Seebeck effect**
  - electric current arising from a temperature difference
  - can in turn lead to regular shot noise
  - Is \(\Delta T\) noise just the shot noise of this thermally induced current?

- **Electrons in a normal junction** [Shein-Lumbroso et al., Nature 562, 240 (‘18)]
  - measure the thermoelectric voltage
  - calculate the associated shot noise
  - orders of magnitude smaller than \(\Delta T\)-noise

- Why it should matter even less here?
  - incompressible bulk
  - chiral edge states, overlapping only at the QPC
  - no equilibration expected over micron-size distances
Experimental realization I

- Measuring backscattered current fluctuations
  - no direct access to the noise $S_B$
  - possible to measure currents $l_3, l_4$ and their auto- and cross-correlations $S_{33}, S_{44} \text{ and } S_{34}$

- From backscattered current to measurable currents
  \[
  \langle l_3(x_3, t) \rangle = -\mathcal{I}_B + \frac{\nu e^2}{2\pi} V
  \]
  \[
  \langle l_4(x_4, t) \rangle = \mathcal{I}_B
  \]

- From backscattered current fluctuations to cross-correlations
  - Cross-correlations
    \[
    S_{34}(t - t') = \langle l_3(x_3, t)l_4(x_4, t') \rangle - \langle l_3(x_3, t) \rangle \langle l_4(x_4, t') \rangle
    \]
  - Connection with backscattered current fluctuations
    \[
    S_{34} = 2 \left( T_R + T_L \right) \frac{\partial \mathcal{I}_B}{\partial V} - S_B
    \]

$\Rightarrow$ $S_B$ can be measured via $S_B = 2(T_L + T_R)G_4 - S_{34}$
Experimental realization II

- Reparametrization

\[
\begin{align*}
T_L &= \bar{T} - \frac{\Delta T}{2} \rightarrow T_L = T_{\text{cold}} \\
T_R &= \bar{T} + \frac{\Delta T}{2} \rightarrow T_R = T_{\text{hot}} = T_{\text{cold}} + \Delta T
\end{align*}
\]

- Three-step measurement

1. Measure at base temperature $T_{\text{cold}}$

2. Heat up one entry port at $T_{\text{hot}}$

3. Heat up the other entry port at $T_{\text{hot}}$

\[
\Delta S_{\text{II}} = S_2 - S_1 = S_{\text{cold}} (2\nu - 1) \frac{\Delta T}{2T_{\text{cold}}} \times \left[ 1 + \left( \nu - 1 + \frac{C^{(2)}_{\nu}}{2\nu - 1} \right) \frac{\Delta T}{2T_{\text{cold}}} \right]
\]

\[
\Delta S_{\text{III}} = S_2 - \frac{1}{2} (S_1 + S_3) = S_{\text{cold}} \left[ C^{(2)}_{\nu} - (2\nu - 1)(\nu - 1) \right] \left( \frac{\Delta T}{2T_{\text{cold}}} \right)^2
\]
Experimental realization II

- Reparametrization

\[
\begin{align*}
T_L &= \bar{T} - \frac{\Delta T}{2} \quad \rightarrow \quad T_L = T_{\text{cold}} \\
T_R &= \bar{T} + \frac{\Delta T}{2} \quad \rightarrow \quad T_R = T_{\text{hot}} = T_{\text{cold}} + \Delta T
\end{align*}
\]

- Three-step measurement

1. \(\rightarrow\) \(S(T_{\text{cold}}, T_{\text{cold}})\) measure at base temperature \(T_{\text{cold}}\)
2. \(\rightarrow\) \(S(T_{\text{hot}}, T_{\text{cold}})\) heat up one entry port at \(T_{\text{hot}}\)
3. \(\rightarrow\) \(S(T_{\text{hot}}, T_{\text{hot}})\) heat up the other entry port at \(T_{\text{hot}}\)

\[
\Delta S_{II} = S_{\bar{I}} - S_{\odot}
= S_{\text{cold}} (2\nu - 1) \left( \frac{\Delta T}{2T_{\text{cold}}} \right) \frac{\Delta T}{2T_{\text{cold}}}
\times \left[ 1 + \left( \nu - 1 + \frac{C_{\nu}^{(2)}}{2\nu-1} \right) \frac{\Delta T}{2T_{\text{cold}}} \right]
\]

\[
\Delta S_{III} = S_{\bar{I}} - \frac{1}{2} (S_{\odot} + S_{\bar{3}})
= S_{\text{cold}} \left[ C_{\nu}^{(2)} - (2\nu - 1)(\nu - 1) \right] \left( \frac{\Delta T}{2T_{\text{cold}}} \right)^2
< 0 \text{ for } \nu = \frac{1}{2n+1}
\]