Disordered one-dimensional bosons

Nicolas Dupuis

Laboratoire de Physique Théorique de la Matière Condensée
Sorbonne Université & CNRS, Paris

based on work with Romain Daviet:

ND, Phys. Rev. E, 2019, 100, 030102(R)
ND & R. Daviet, Phys. Rev. E, 2020, 101, 042139
ND, Europhys. Lett., 2020, 130, 56002
R. Daviet & ND, Phys. Rev. Lett. 125, 235301 (2020)
R. Daviet & ND, Phys. Rev. E 103, 052136 (2021)

Outline

- Introduction: disordered quantum systems

  localized phases and glassy properties

- Disordered 1D bosons
  - bosonization and replica formalism
  - perturbative RG: Bose-glass phase
  - functional RG: strong-disorder fixed point, metastable states and glassy properties of the BG
- Mott-glass phase due to long-range interactions
- Conclusion

Disordered quantum systems

- Single particle in a random potential: Anderson localization

  the wavefunction is localized if the disorder is strong enough or in sufficiently low dimension.

- Many-particle quantum systems: the interplay between disorder and interactions (in general a complicated problem) may lead to a localization transition.

- The localized phase is characterized by a vanishing dc conductivity.

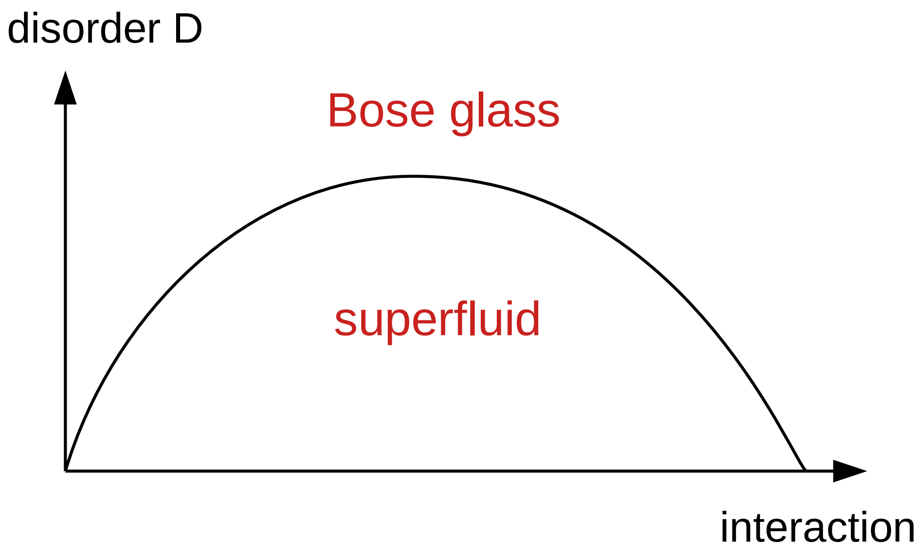
  As in classical systems, one also expects “glassy” properties due to the existence of metastable states: non-ergodicity, pinning and “shocks”, depinning transition and avalanches, chaotic behavior, slow dynamics, etc.

1D Bose fluid

- Pure fluid
  - g = 0 (no interaction) : BEC
  - g ≠ 0 : superfluid state (finite phase stiffness) without BEC (Luttinger liquid)

- Disordered fluid
  - a quantum particle in a 1D random potential: the wave function is localized (Anderson localization)
  - g = 0 : BEC in the lowest-energy state
  - g ≠ 0 : superfluid to Bose-glass transition

    [Giamarchi & Schulz'87, '88, Fisher *et al.*'89]



One-dimensional Bose fluid

- Hamiltonian $$H = \int dx \;\; \psi^\dagger(x) \left(-\frac{\partial_x^2}{2m} - \mu \right) \psi(x) + g\big(\psi^\dagger(x)\psi(x)\big)^2$$

- bosonization [Haldane 1981]

$$\psi(x) = e^{i\theta(x)} \sqrt{\rho(x)}$$
$$\rho(x) = \rho_0 - \frac{1}{\pi}\partial_x \varphi(x) + 2\rho_2 \cos(2\pi\rho_0 x + 2\varphi(x)) + \cdots$$

$$[\theta(x), \partial_y \varphi(y)] = i\pi\delta(x-y)$$

- Luttinger liquid: superfluid state without BEC

$$H_{\mathrm{LL}} = \int dx \frac{v}{2\pi} \left\{ K(\partial_x\theta)^2 + \frac{1}{K}(\partial_x\varphi)^2 \right\}$$

$$S_{\mathrm{LL}} = \int dx \int_0^\beta d\tau \frac{v}{2\pi K} \left\{ (\partial_x\varphi)^2 + \frac{(\partial_\tau\varphi)^2}{v^2} \right\}$$

Interacting bosons in a random potential

- action

$$S = S_{\rm LL} + \int_{x,\tau} V(x)\rho(x,\tau) \quad \text{with} \quad \left\{ \begin{array}{l} \overline{V(x)} = 0 \\ \overline{V(x)V(x')} = D\delta(x-x') \end{array} \right.$$

- Replica formalism: *n* copies of the system

$$Z = \overline{\prod_{a=1}^{n} Z[V]} = \int \mathcal{D}[\{\varphi_a\}]\, e^{-S[\{\varphi_a\}]}$$

with "replicated" action ("sine-Gordon" model)

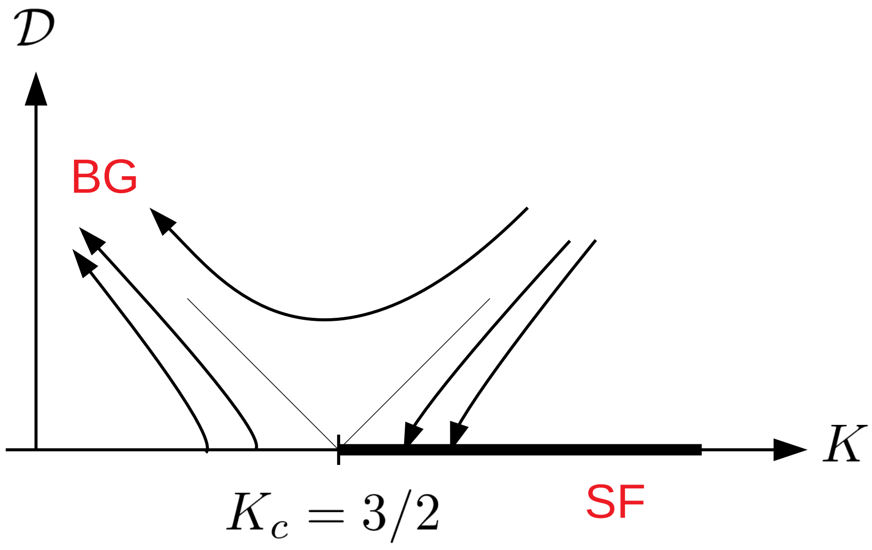
$$S[\{\varphi_a\}] = \sum_{a=1}^{n} \int dx \int_0^\beta d\tau \frac{v}{2\pi K} \left\{ (\partial_x \varphi_a)^2 + \frac{(\partial_\tau \varphi_a)^2}{v^2} \right\}$$
$$-\mathcal{D} \sum_{a,b=1}^{n} \int dx \int_0^\beta d\tau\, d\tau' \cos[2\varphi_a(x,\tau) - 2\varphi_b(x,\tau')]$$

Perturbative RG [Giamarchi, Schulz 1988, Ristivojevic *et al.* 2012]

- phase diagram

$$\frac{dK}{dl} = -K^2 \frac{\mathcal{D}}{\pi v^2}$$
$$\frac{d\mathcal{D}}{dl} = (3 - 2K)\mathcal{D}$$
$$\frac{d}{dl}\left(\frac{v}{K}\right) = 0$$

(BKT flow)



- Bose-glass phase [Fisher *et al.* 1989]

compressibility: $d\kappa/dl = 0, \quad \kappa > 0$

localized phase: $\xi_{\text{loc}} \sim \mathcal{D}^{-\frac{1}{3-2K}}$

gapless conductivity: $\sigma(\omega)$

Functional renormalization group

$$S[\{\varphi_a\}] = \sum_{a=1}^{n} \int dx \int_0^\beta d\tau \frac{v}{2\pi K} \left\{ (\partial_x \varphi_a)^2 + \frac{(\partial_\tau \varphi_a)^2}{v^2} \right\} - \int dx \int_0^\beta d\tau \, d\tau' \sum_{a,b=1}^{n} \underbrace{\mathcal{D} \cos[2\varphi_a(x,\tau) - 2\varphi_b(x,\tau')]}$$

Renormalized disorder correlator $V(\varphi_a(x,\tau) - \varphi_b(x,\tau'))$

- classical disordered systems: the functional disorder correlator $V(\varphi_a - \varphi_b)$ may assume a non-analytic "cuspy" form that encodes the metastable states of the system and the ensuing glassy properties: pinning, "shocks" and "avalanches", chaotic behavior, aging, etc.

- Long history in classical disordered systems… Fisher 1985, Narayan, Balents, Nattermann, Chauve, Le Doussal, Wiese, etc.

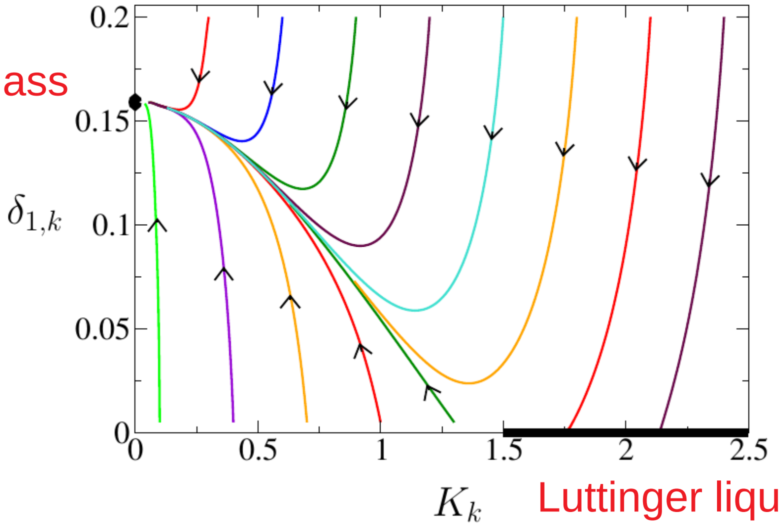
- non-perturbative (Wetterich's) formulation: Tissier & Tarjus 2004- (RFIM)

[ND *et al.*, Phys. Rep. 2021, *The nonperturbative functional renormalization group and its applications*]

- phase diagram

$$\delta_k(u) = -\frac{K^2}{v^2}\frac{V_k''(u)}{k^3} = \sum_{n=1}^{\infty} \delta_{n,k}\cos(2nu)$$

Bose glass

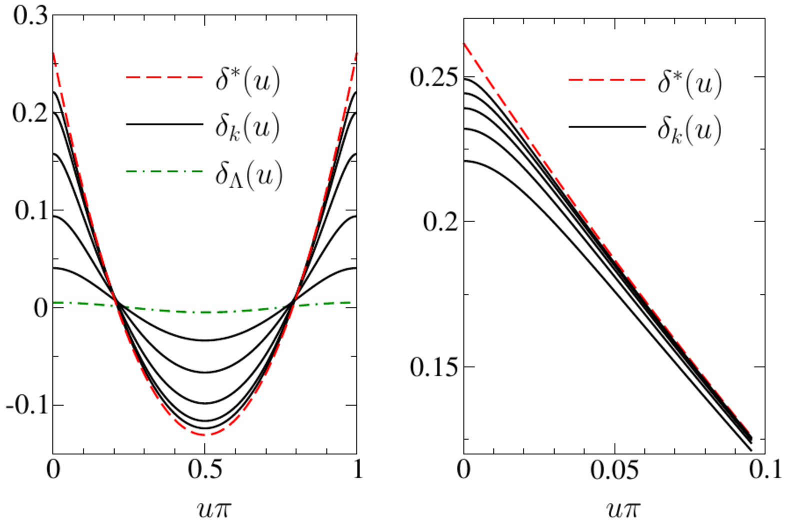


Luttinger liquid

- Bose-glass fixed point:

$$K^* = 0, \quad K_k \sim k^\theta \quad \text{no quantum fluctuations, hence pinning}$$

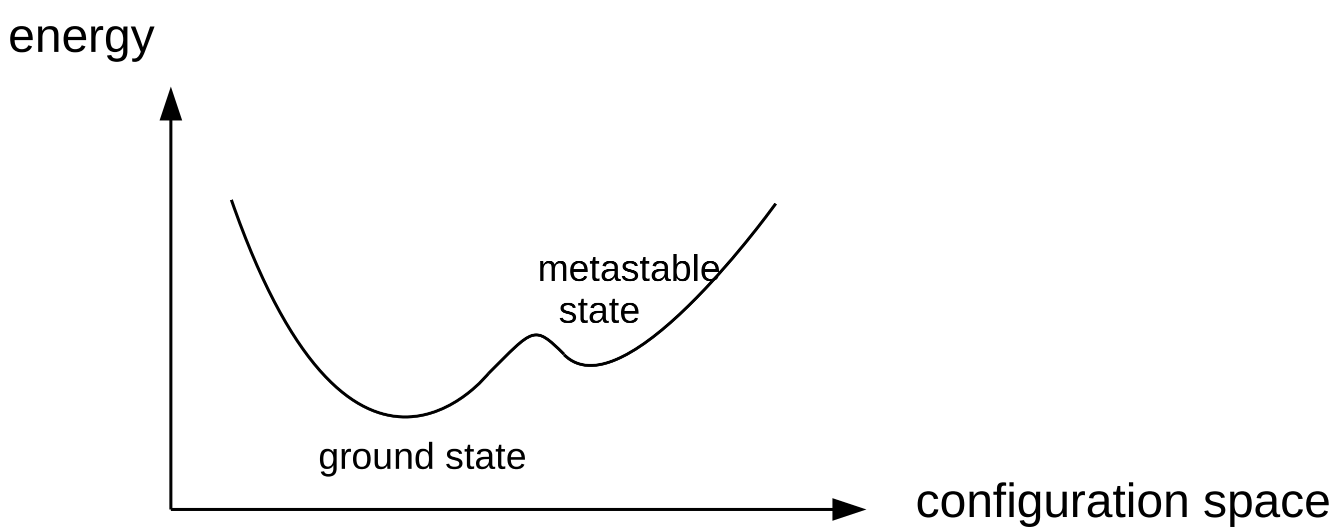
$$\delta^*(u) = -\frac{K^2}{v^2}\lim_{k\to 0}\frac{V_k''(u)}{k^3} = \frac{1}{2a_2}\left[\left(u-\frac{\pi}{2}\right)^2 - \frac{\pi^2}{12}\right] \quad \text{for} \quad u\in[0,\pi]$$



cusp
and quantum boundary layer
(controlled by $K_k \sim k^\theta$)

Physics of the cusp and the boundary layer: metastable states

[Balents *et al.* 1996, Le Doussal, etc.]



- cusp: the (classical) ground state varies discontinuously, as a function of external parameters, whenever it becomes degenerate with a metastable state: “shocks” or “avalanches”.
- quantum boundary layer: quantum fluctuations (K>0) lead to quantum tunneling between nearly degenerate states and a rounding of the cusp in a boundary layer.
- the low-energy physics is dominated by the (quantum-mechanically active) classical metastable states (see the “droplet picture” put forward by Fisher and Huse (1988) for disordered classical systems), e.g.

$$\sigma(\omega) \sim \omega^2$$

in agreement with hard-core bosons and free fermions (K=1): $\sigma(\omega) \sim \omega^2 \ln^2 \omega$

1D Bose fluid with long-range interactions [R. Daviet & ND, PRL 2020]

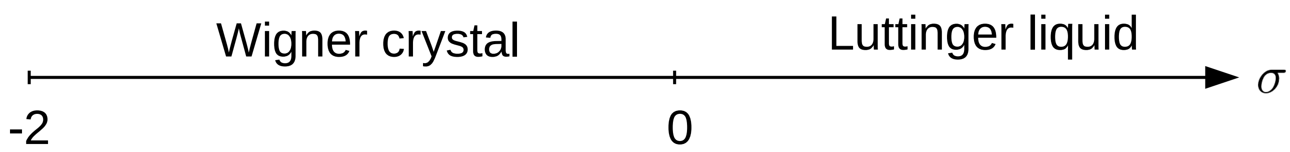
- Mott glass vs Bose glass

	Bose glass	Mott glass	Mott insulator
compressibility	>0	0	0
conductivity	ω^2	ω^2	0

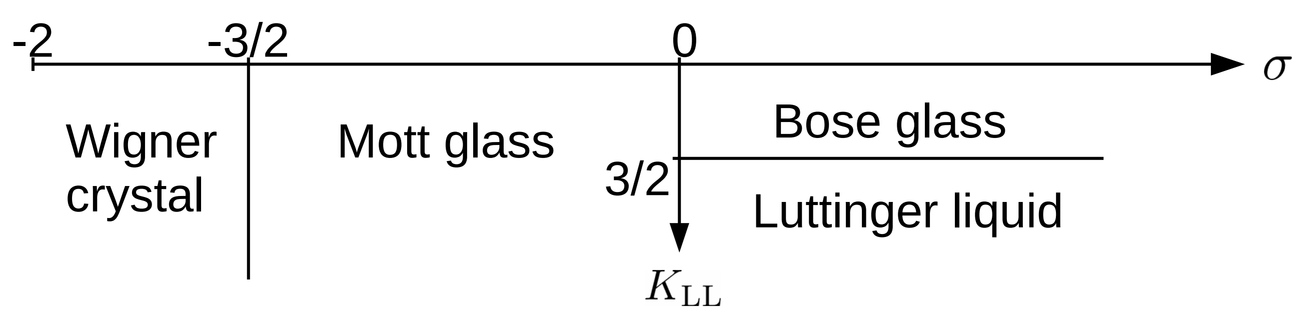
$$V_\sigma(x) = \begin{cases} \frac{e^2}{|x|^{1+\sigma}} & \text{if} \quad \sigma > -1 \\ -e^2 \ln|x/a| & \text{if} \quad \sigma = -1 \\ -e^2 |x|^{-1-\sigma} & \text{if} \quad -2 \leq \sigma < -1 \end{cases} \qquad \text{i.e.} \quad V_\sigma(q) \sim \begin{cases} -\ln|qa| & \text{if} \quad \sigma = 0 \\ |q|^\sigma & \text{if} \quad \sigma < 0 \end{cases}$$

$$\sigma = 0: \ \text{Coulomb}, \qquad \sigma = -2: \ \text{Schwinger model}$$

- pure fluid



- disordered fluid



Conclusion

- The non-perturbative FRG is a powerful method to study the disordered 1D Bose fluid.

- FRG gives a fairly complete picture of the Bose-glass phase and reveals (some of) its glassy properties: pinning, metastable states, “shocks”, etc.

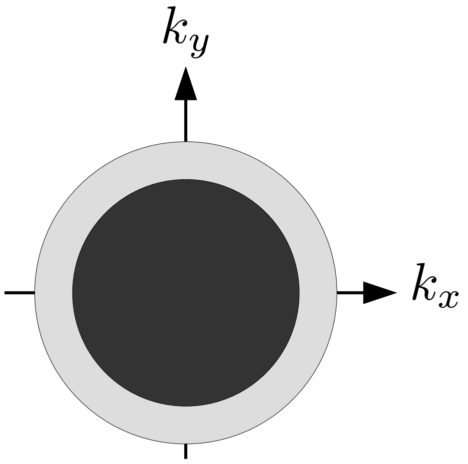
  Many of these results are similar to those obtained in classical disordered systems in which the long-distance physics is controlled by a zero-temperature fixed point.

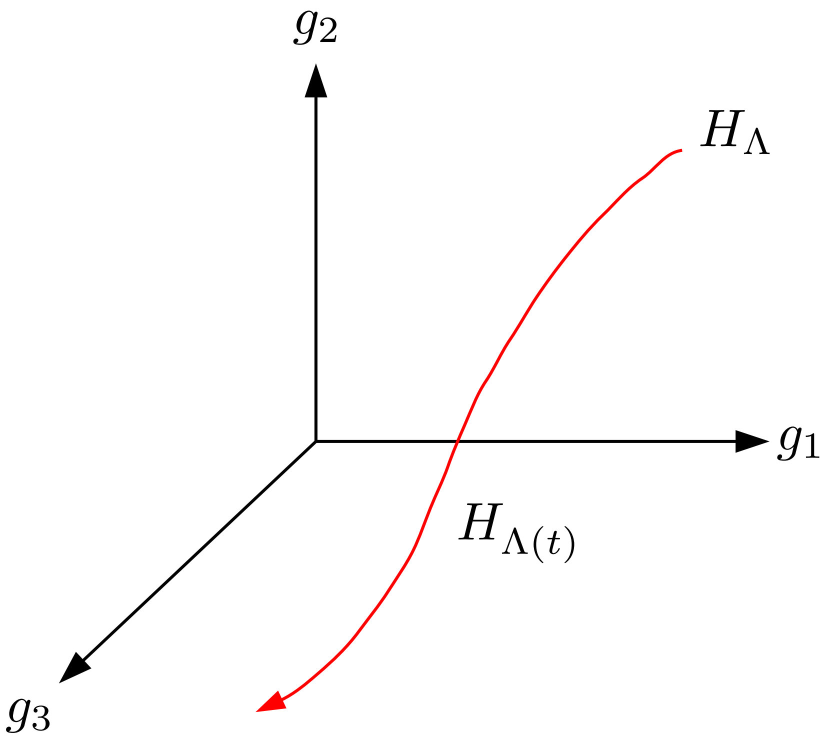
- Long-range interactions can stabilize a Mott-glass phase.

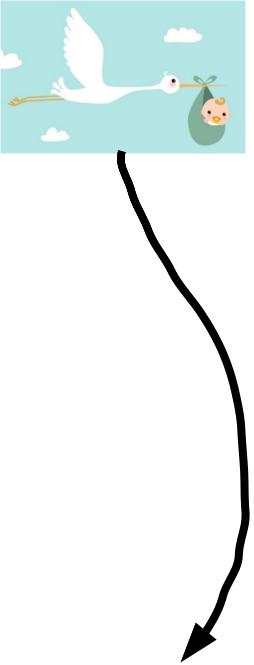
Renormalization group

$$Z = \mathrm{Tr}\, e^{-\beta H}$$

$$\Lambda\, e^{-t} \leq |\mathbf{k}| \leq \Lambda$$







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Non-running
fixed point

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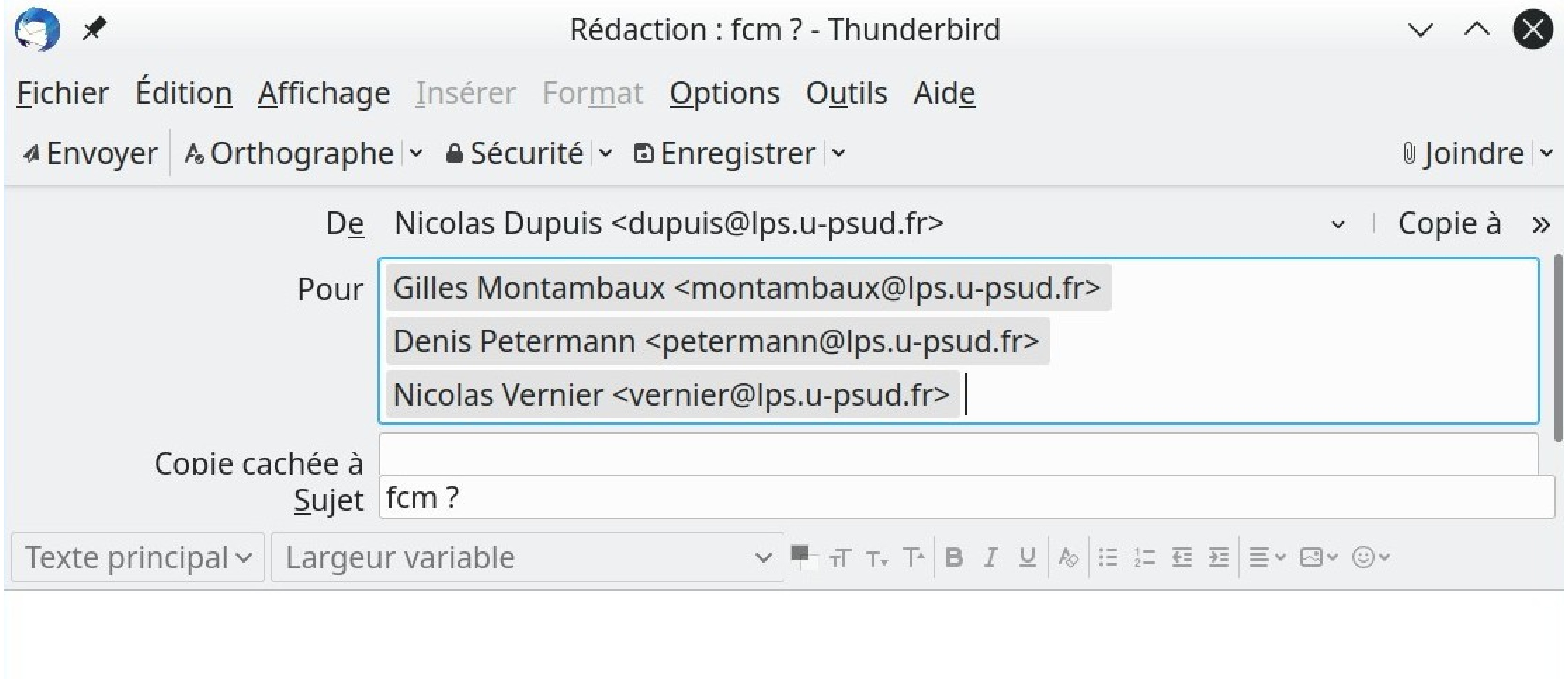
$$H_0 = \sum_k \epsilon_k c_k^\dagger c_k$$

Non-running
fixed point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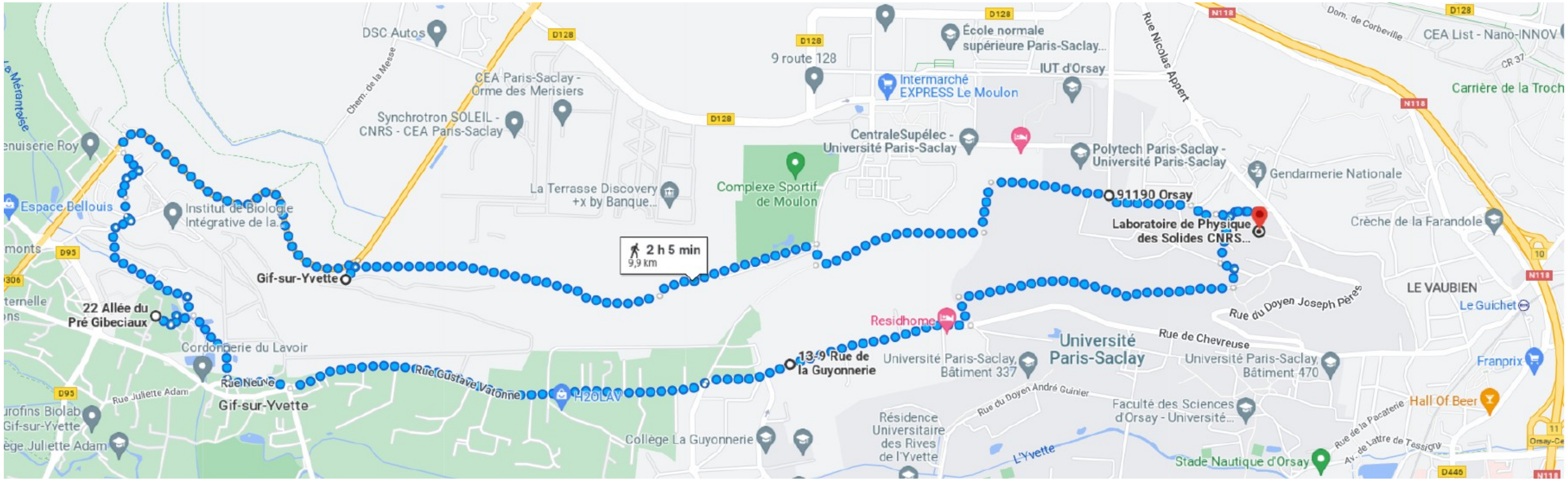
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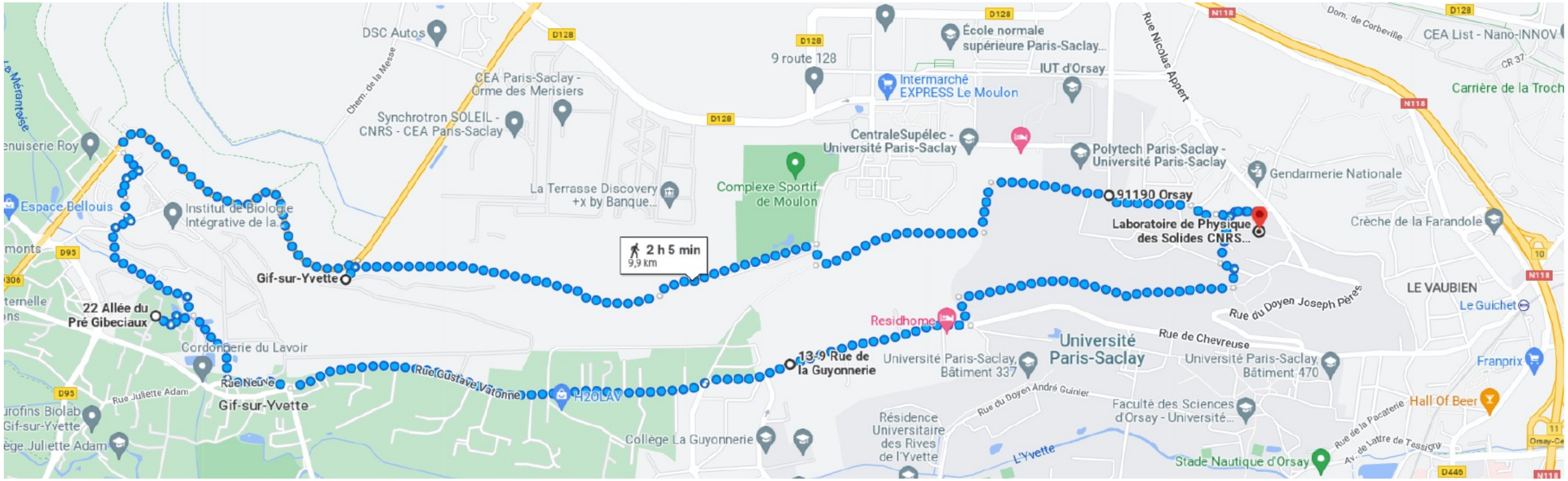
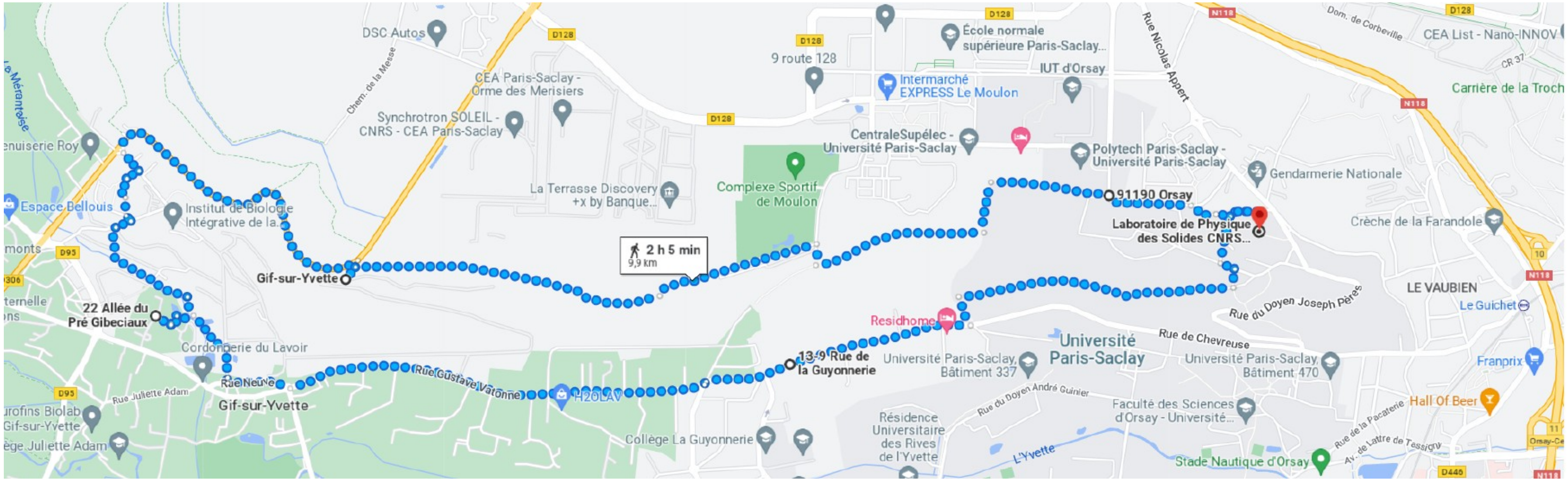


10 km
D+ = 110 m

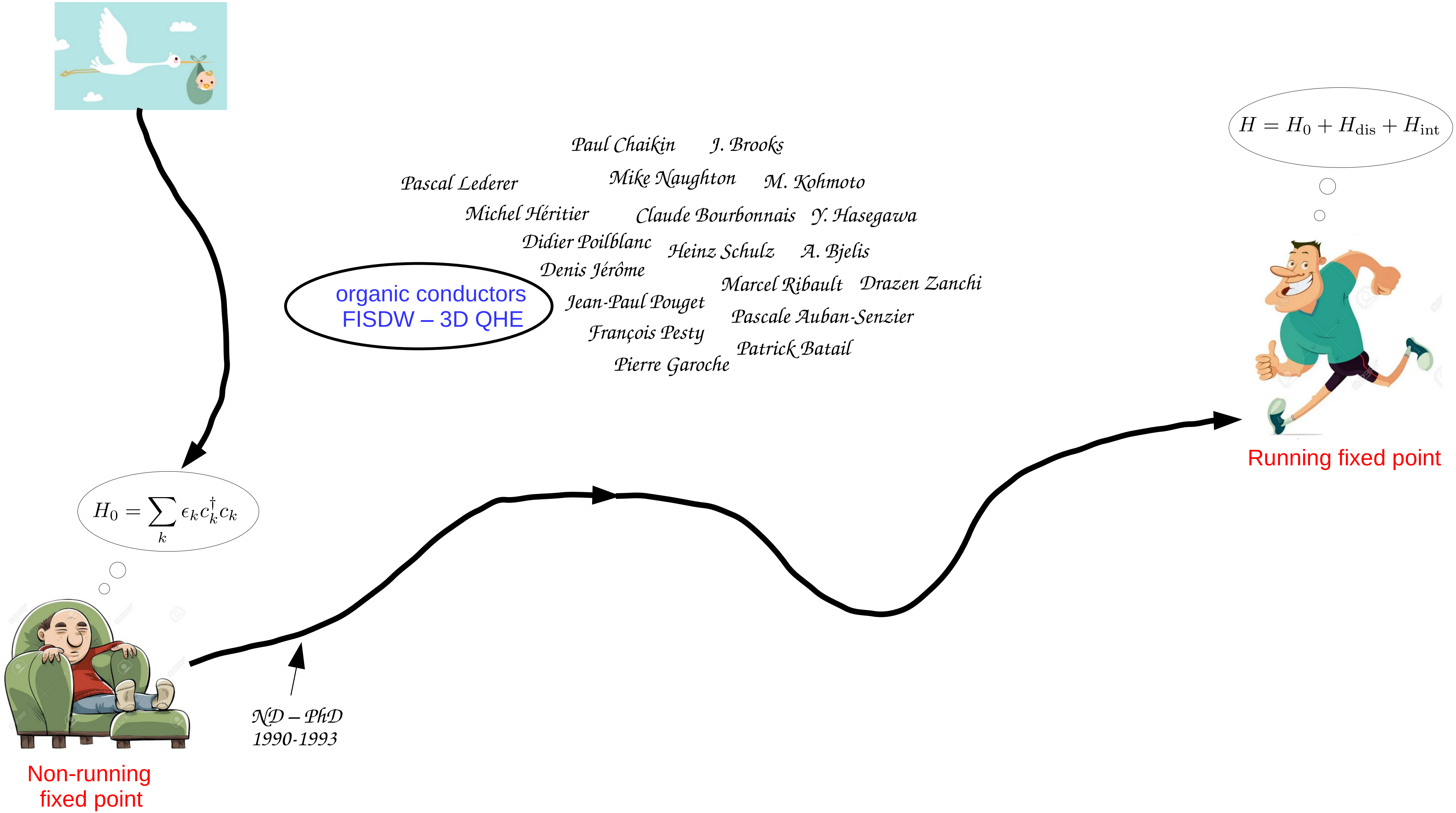
UTMB : 170 km
D+ = 10 000 m



10 km
D+ = 110 m

*2/sem *30sem : 600 km
D+ = 6600 m

UTMB : 170 km
D+ = 10 000 m



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Non-running
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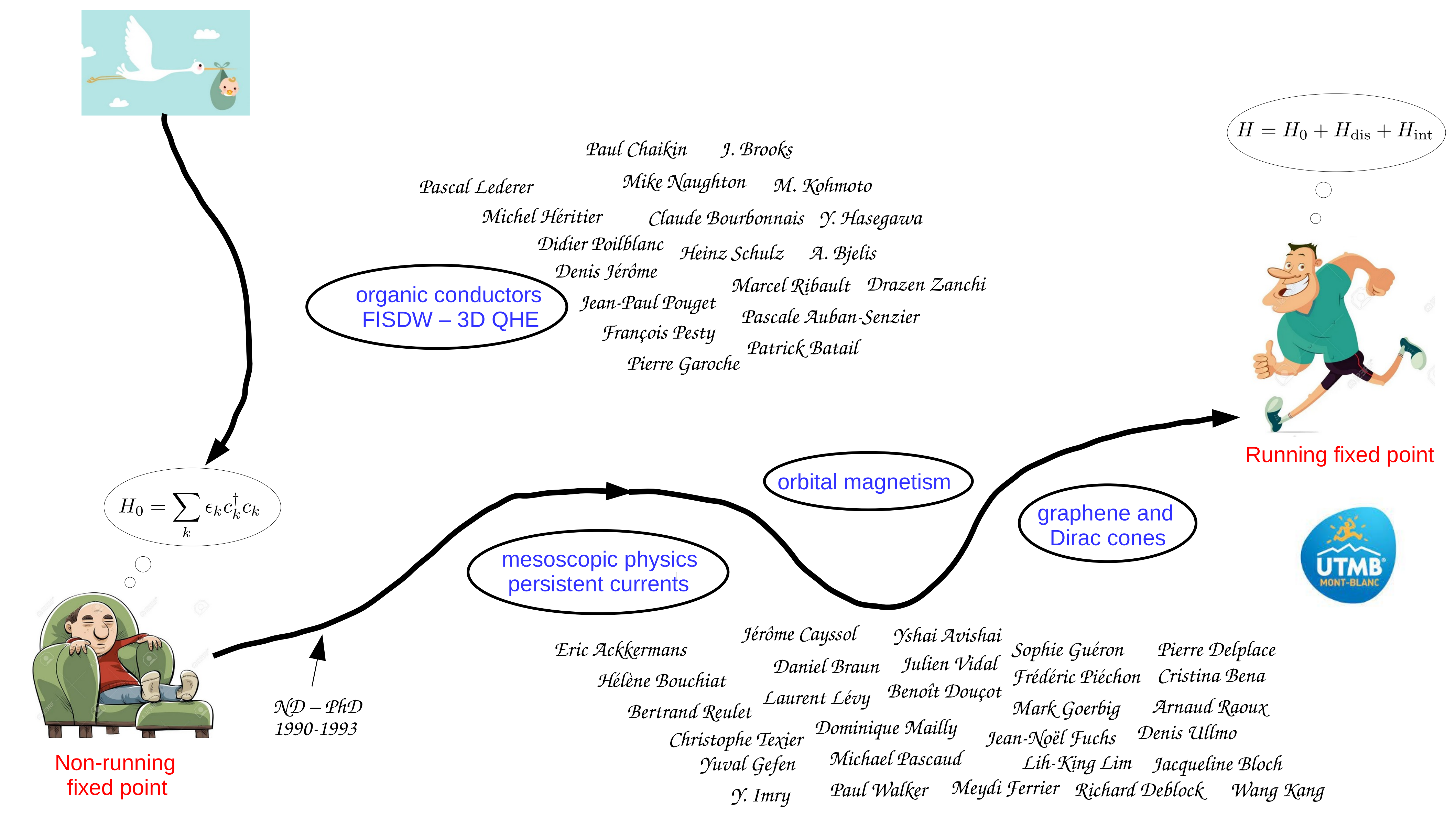

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67
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