

Veselago lensing in Dirac materials

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Outline

I will talk about **Veselago lenses** (or **negative refraction**) for :

1) **The electromagnetic field** (photons)



2) **Electrons** in graphene pn junctions

3) **Electrons and holes** at graphene/superconductor interfaces

4) **Chiral electrons** in 3D Weyl semimetals

I) Veselago lensing with photons

SOVIET PHYSICS USPEKHI

VOLUME 10, NUMBER 4

JANUARY-FEBRUARY 1968

538.30

*THE ELECTRODYNAMICS OF SUBSTANCES WITH SIMULTANEOUSLY NEGATIVE
VALUES OF ϵ AND μ*

V. G. VESELAGO

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Usp. Fiz. Nauk 92, 517–526 (July, 1964)

Electromagnetic waves in matter

Dispersion of an electromagnetic wave:

$$\mathbf{k}^2 = \frac{\omega^2}{c^2} \epsilon_r \mu_r$$

Optical index $n^2 = \epsilon_r \mu_r$

Veselago (1968) considered an hypothetical material where both dielectric constant and magnetic permeability are negative (in some frequency window)

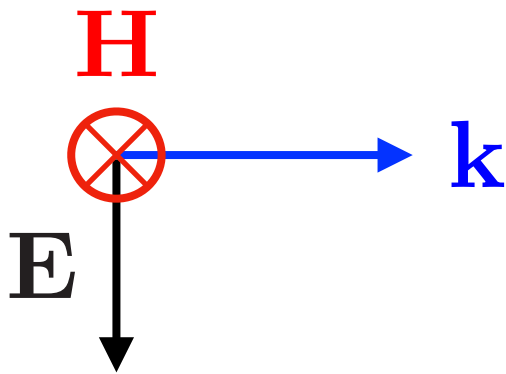
Veselago electrodynamics

$$\mathbf{k} \wedge \mathbf{E} = \omega \mu_0 \mu_r \mathbf{H}$$

$$\mathbf{k} \wedge \mathbf{H} = -\omega \epsilon_0 \epsilon_r \mathbf{E}$$

Standard situation :

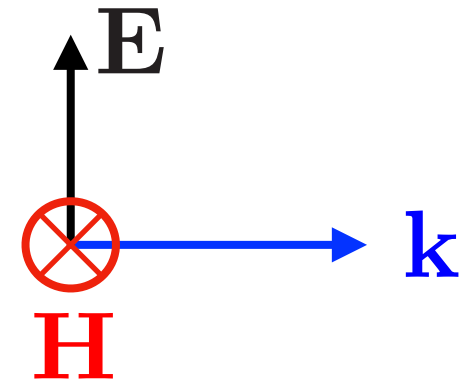
$$\epsilon_r > 0 \quad \mu_r > 0$$



(k,E,H) direct

Veselago situation :

$$\epsilon_r < 0 \quad \mu_r < 0$$



(k,E,H) indirect

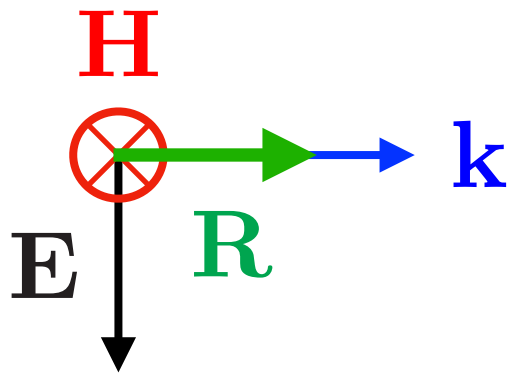
Veselago electrodynamics

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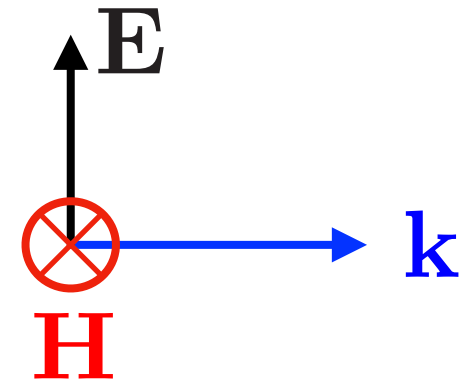
Standard situation :

$$\epsilon_r > 0 \quad \mu_r > 0$$



Veselago situation :

$$\epsilon_r < 0 \quad \mu_r < 0$$



$$\mathbf{R} = \mathbf{E} \wedge \mathbf{H}$$

Poynting vector oriented as : \mathbf{k}

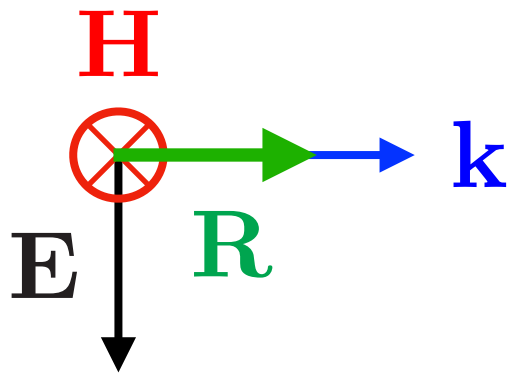
Veselago electrodynamics

$$\mathbf{k} \wedge \mathbf{E} = \omega \mu_0 \mu_r \mathbf{H}$$

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Standard situation :

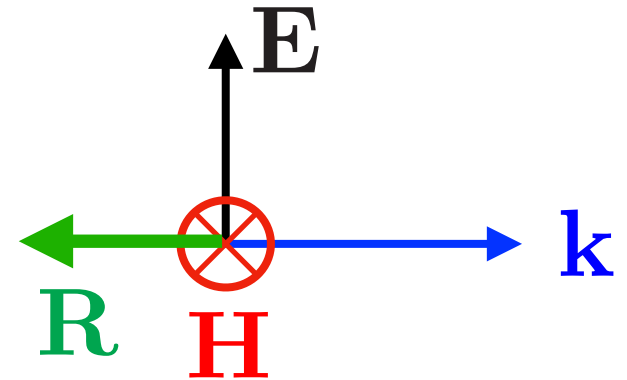
$$\epsilon_r > 0 \quad \mu_r > 0$$



$$\mathbf{R} = \mathbf{E} \wedge \mathbf{H}$$

Veselago situation :

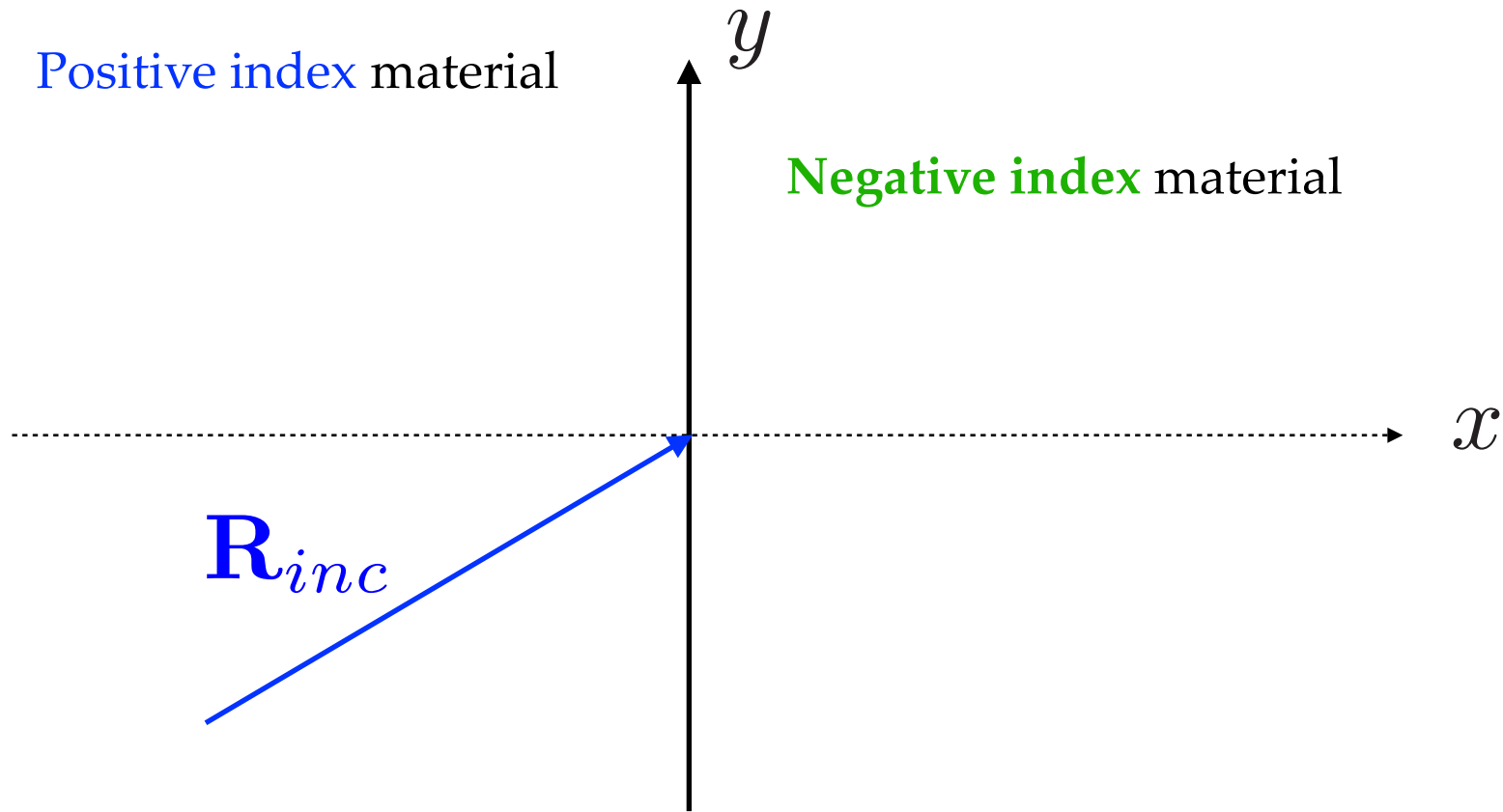
$$\epsilon_r < 0 \quad \mu_r < 0$$



Poynting vector oriented as : $-\mathbf{k}$

Poynting vector oriented as : \mathbf{k}

Negative refraction of rays



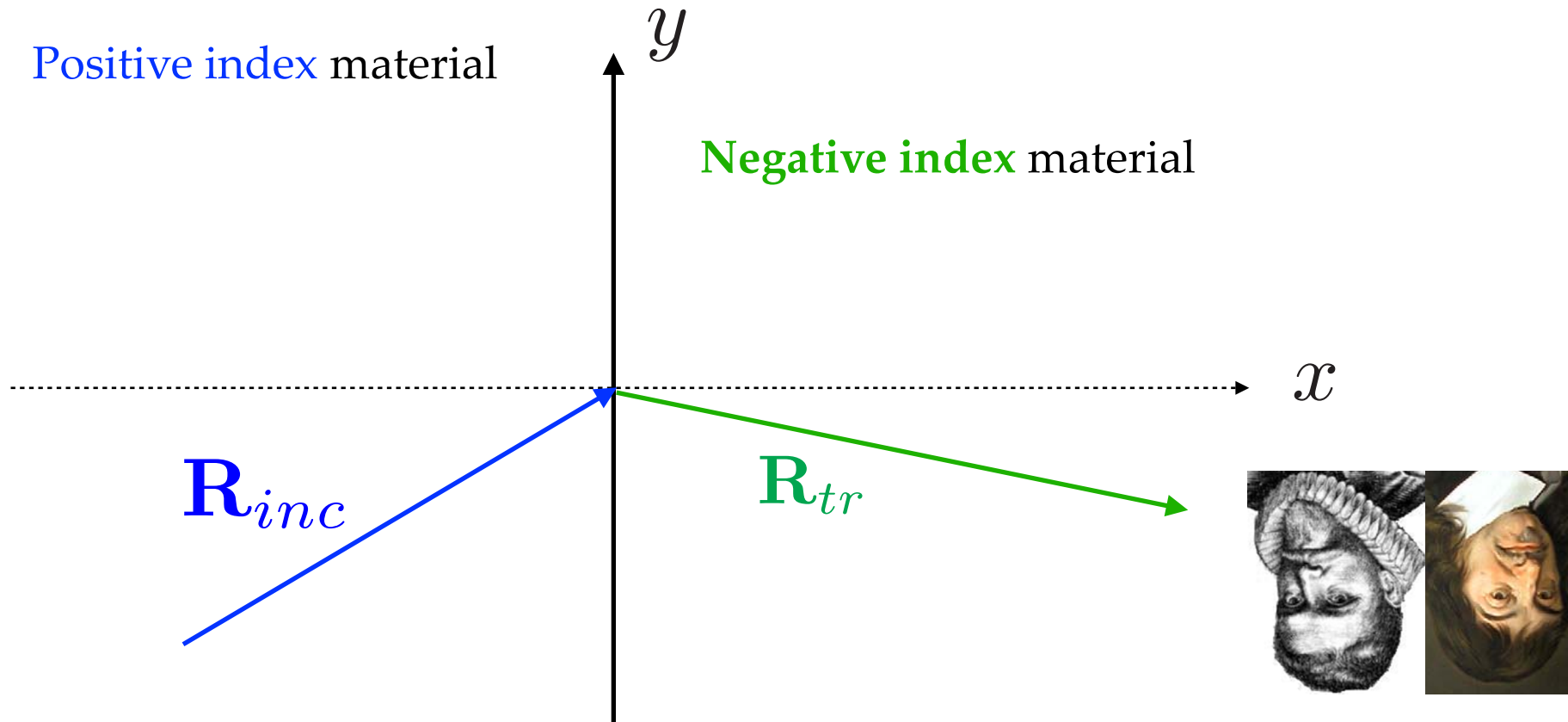
Conservation of
interfacial wave vector

$$\mathbf{e}_y \cdot \mathbf{k}_{inc} = \mathbf{e}_y \cdot \mathbf{k}_{tr} > 0$$

Outgoing-wave

$$\mathbf{e}_x \cdot \mathbf{R}_{tr} > 0$$

Negative refraction of rays



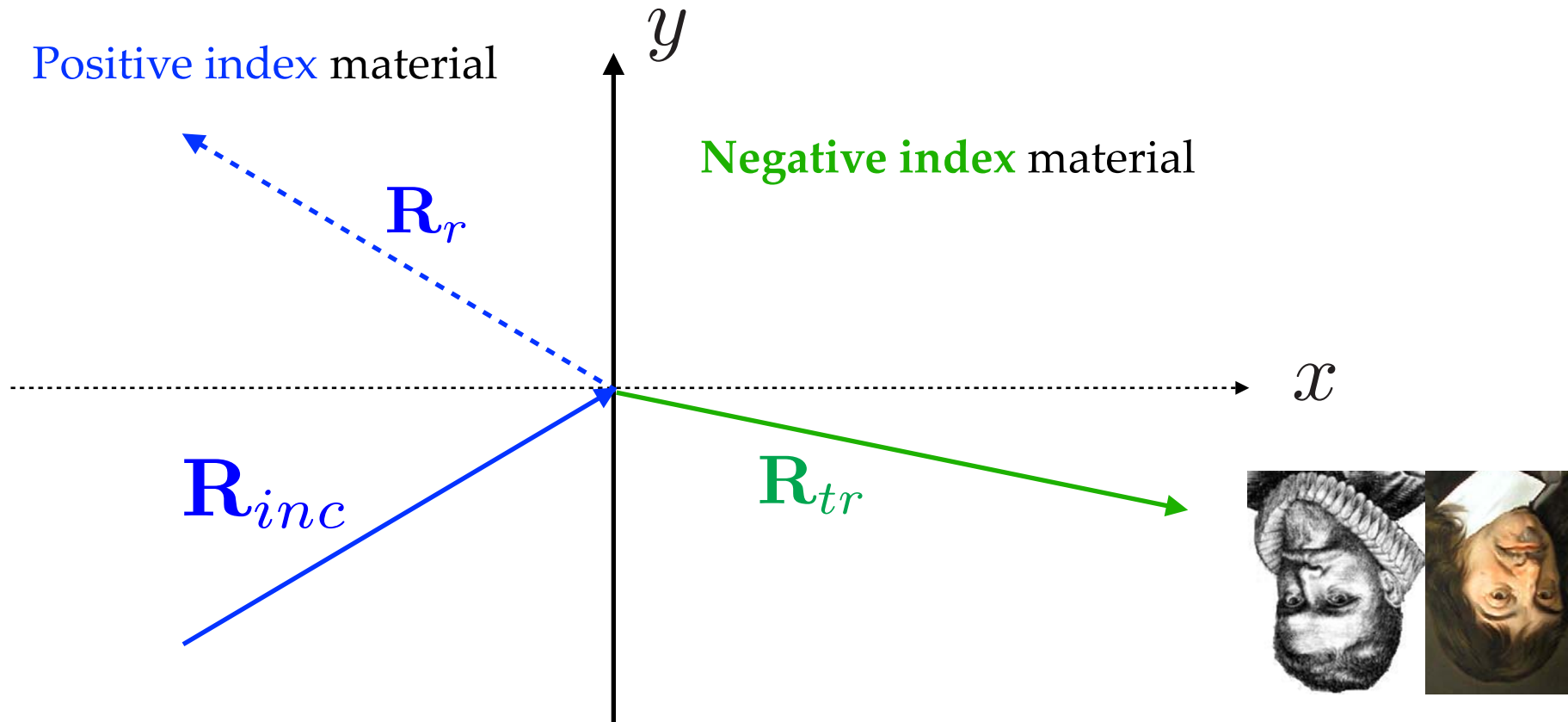
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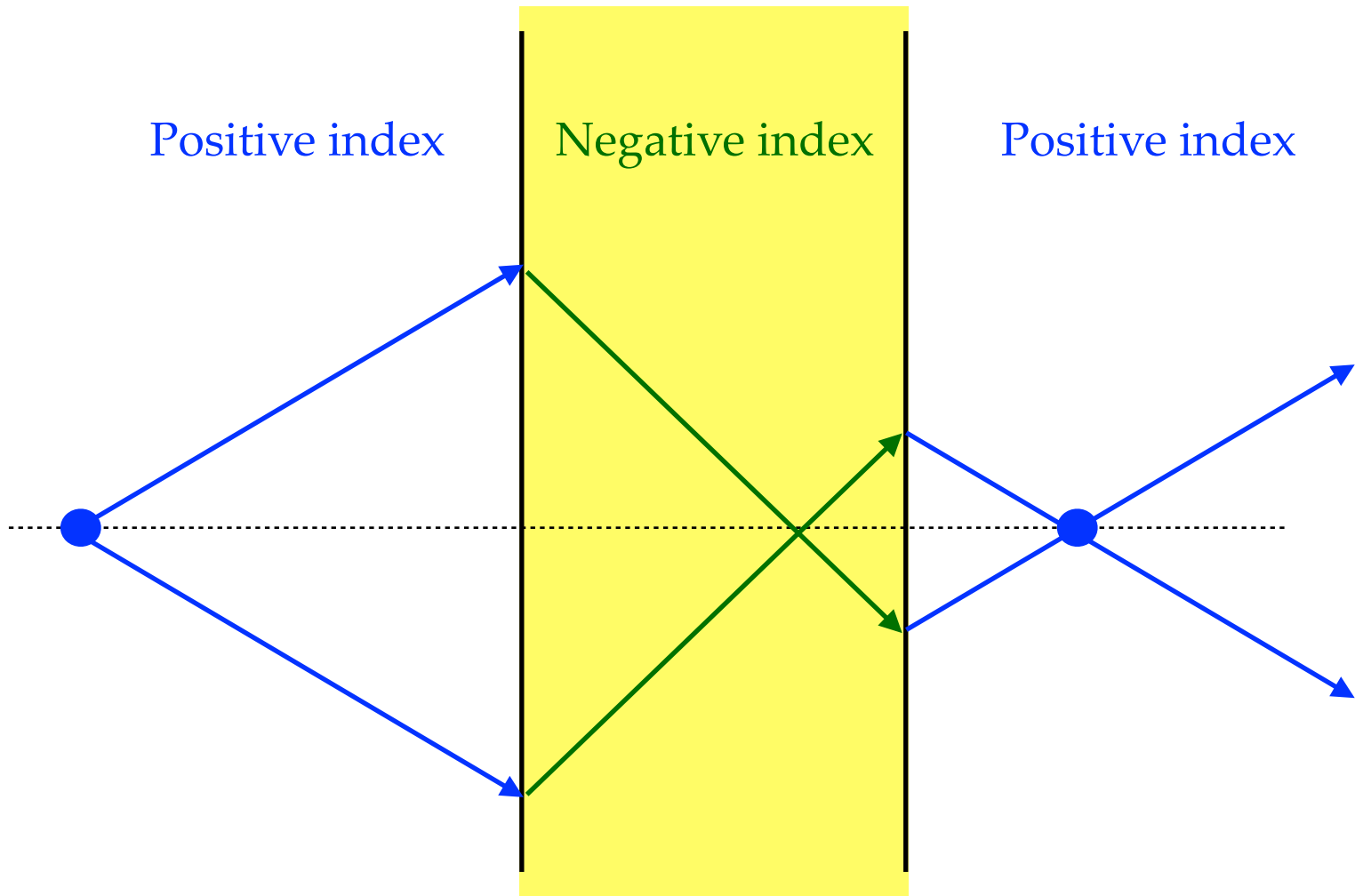
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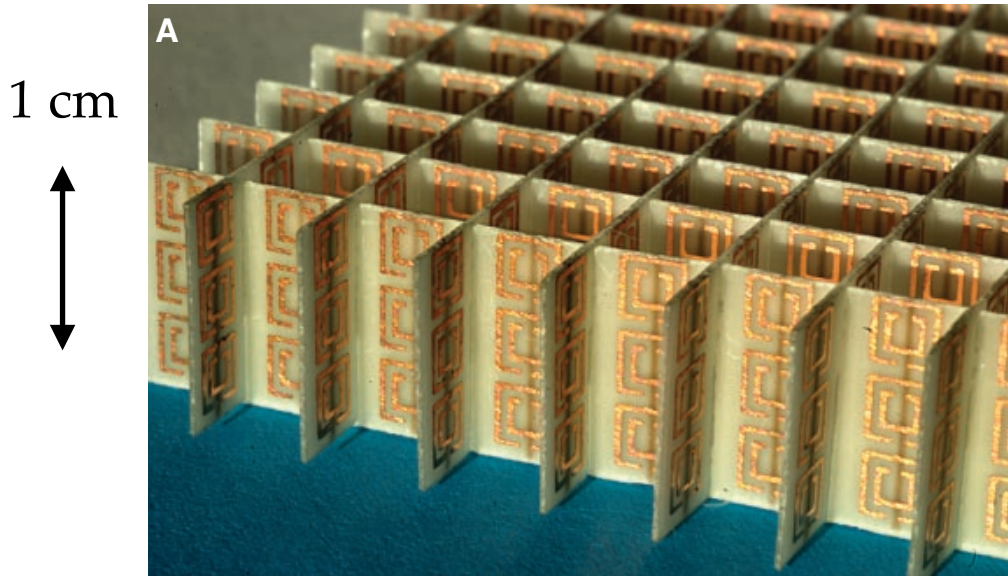
Flat lens focusing



Even a single interface can focus a beam

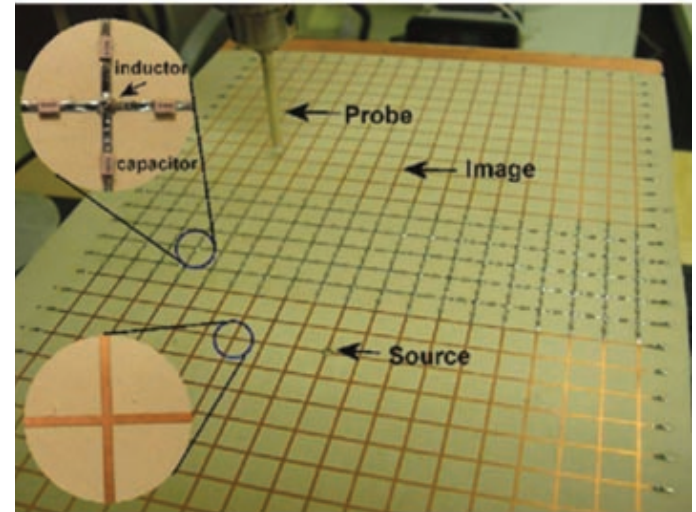
Experiments: metamaterials

Artificial materials with subwavelengths structures



3D metamaterial
Smith (UC San Diego)

Elementary blocks = metallic resonators



2D version of a flat lens

D.R. Smith, J.B. Pendry and M.C.K. Wiltshire, Science 2004

Optics with electrons

Photons

Dispersion

Maxwell equations

Poynting vector

Optical rays

Massless bosons

Non interacting

3D

Polarizations

Electrons in ballistic regime

Electronic band structure

Schrödinger equation

Group velocity

Semiclassical trajectories of electrons

Massive fermions with charge e

e-e interactions

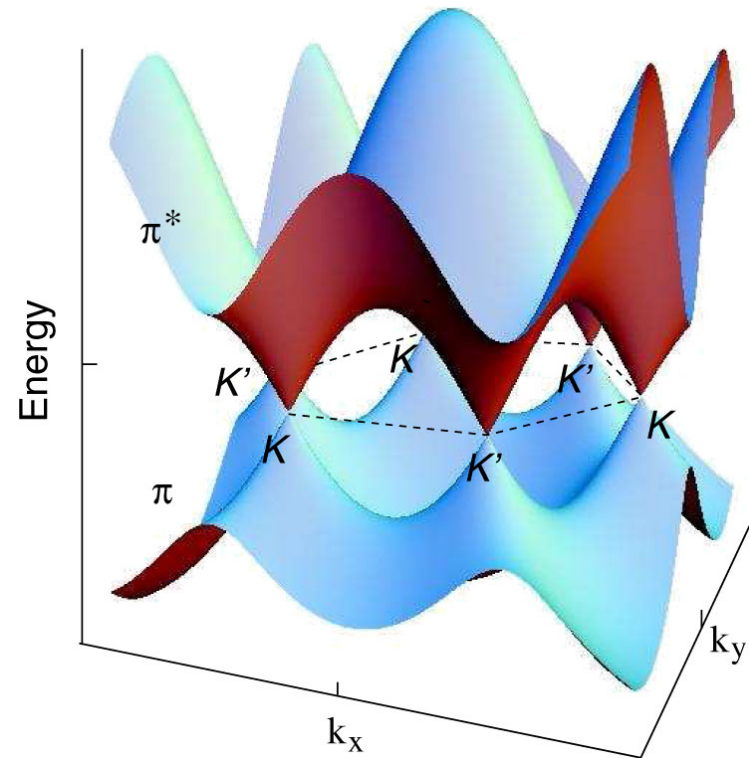
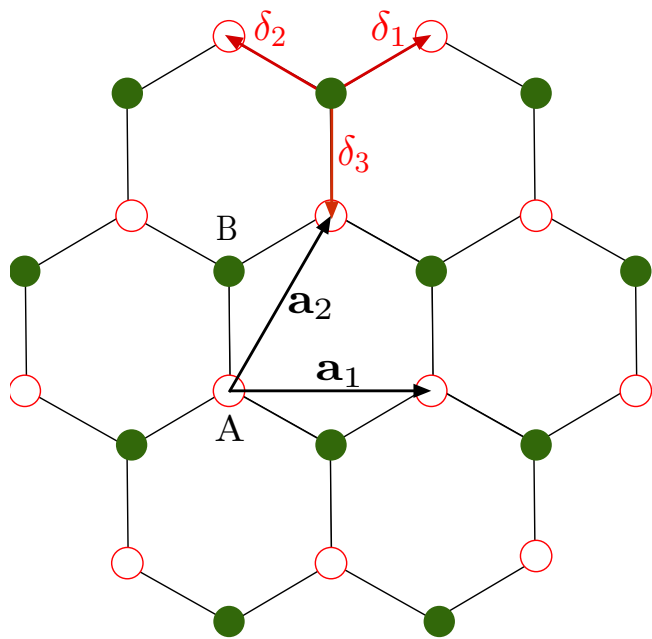
Quasiparticles : 3D, 2D, 1D

Spin

Suitable devices : Electronic lenses, beamsplitters, interferometers for electrons

II) Veselago lensing with massless electrons in 2D graphene

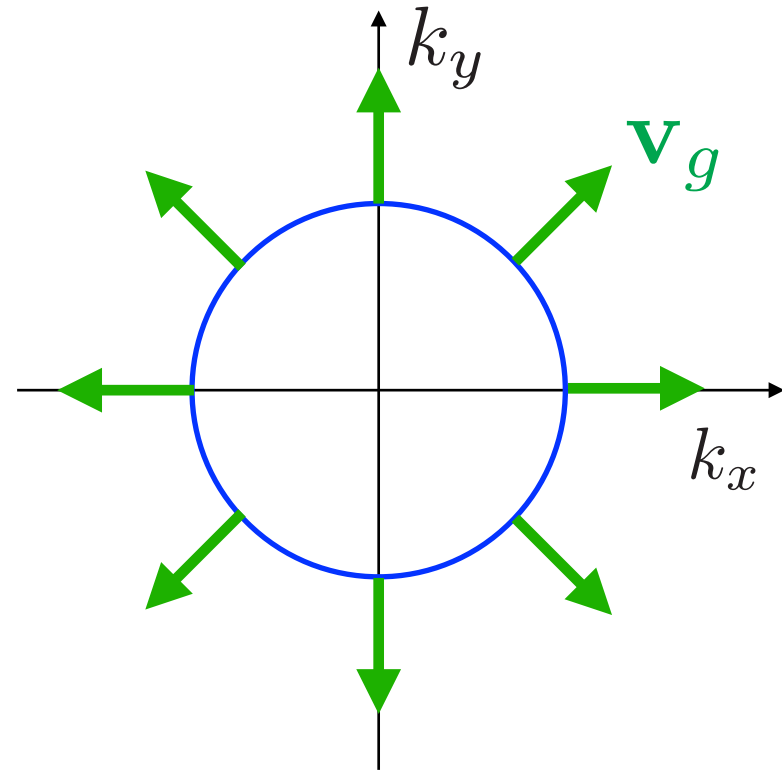
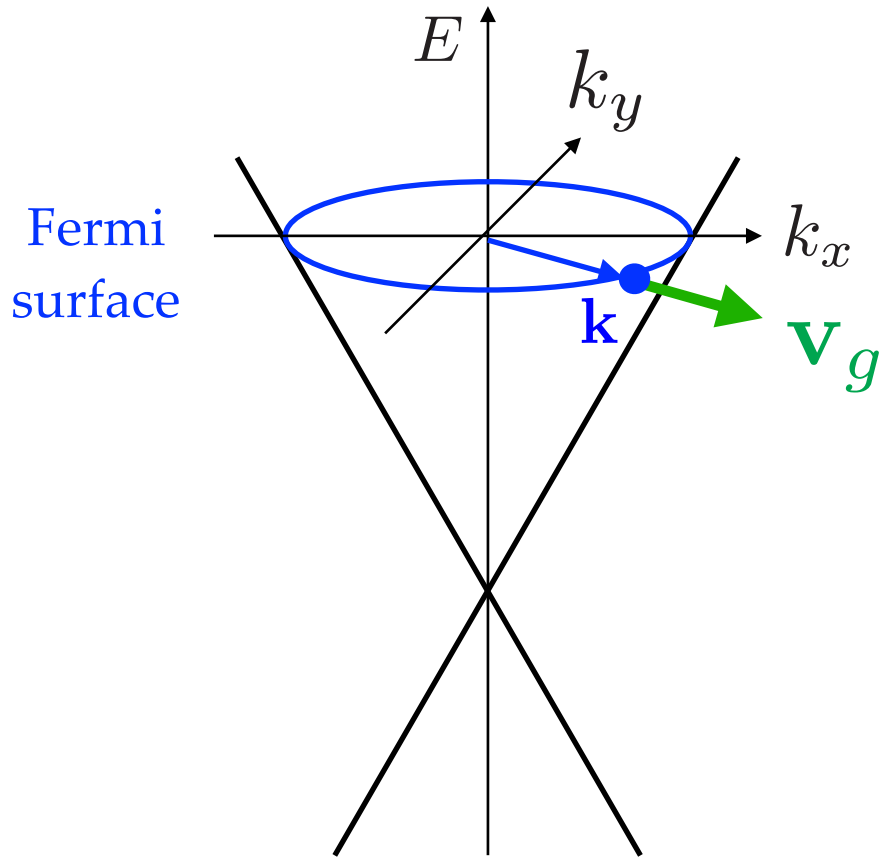
Graphene band structure



$$\mathbf{v}_g = \frac{1}{\hbar} \nabla_{\mathbf{k}} E(\mathbf{k})$$

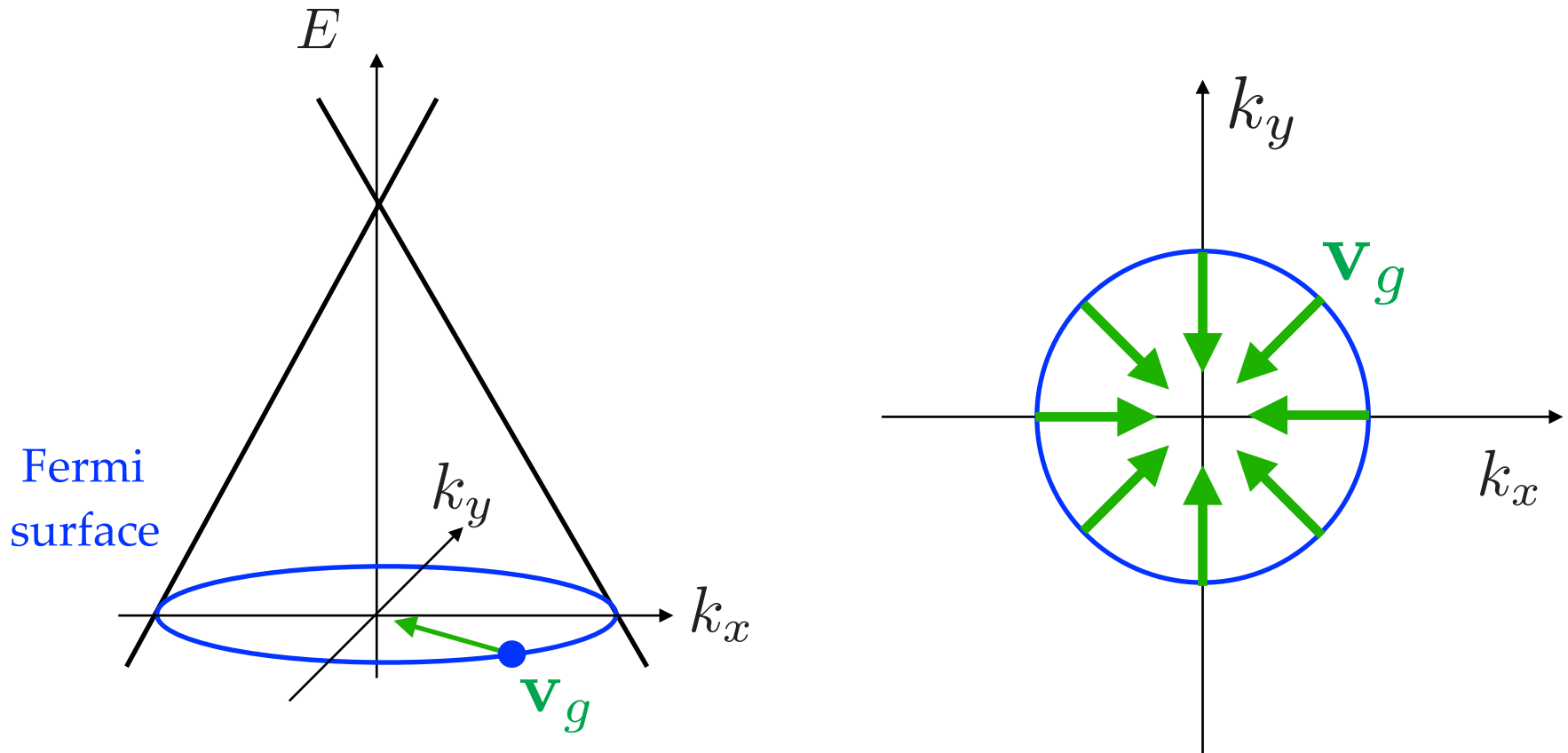
The group velocity is the gradient of the (rich) dispersion relation

n-doped graphene



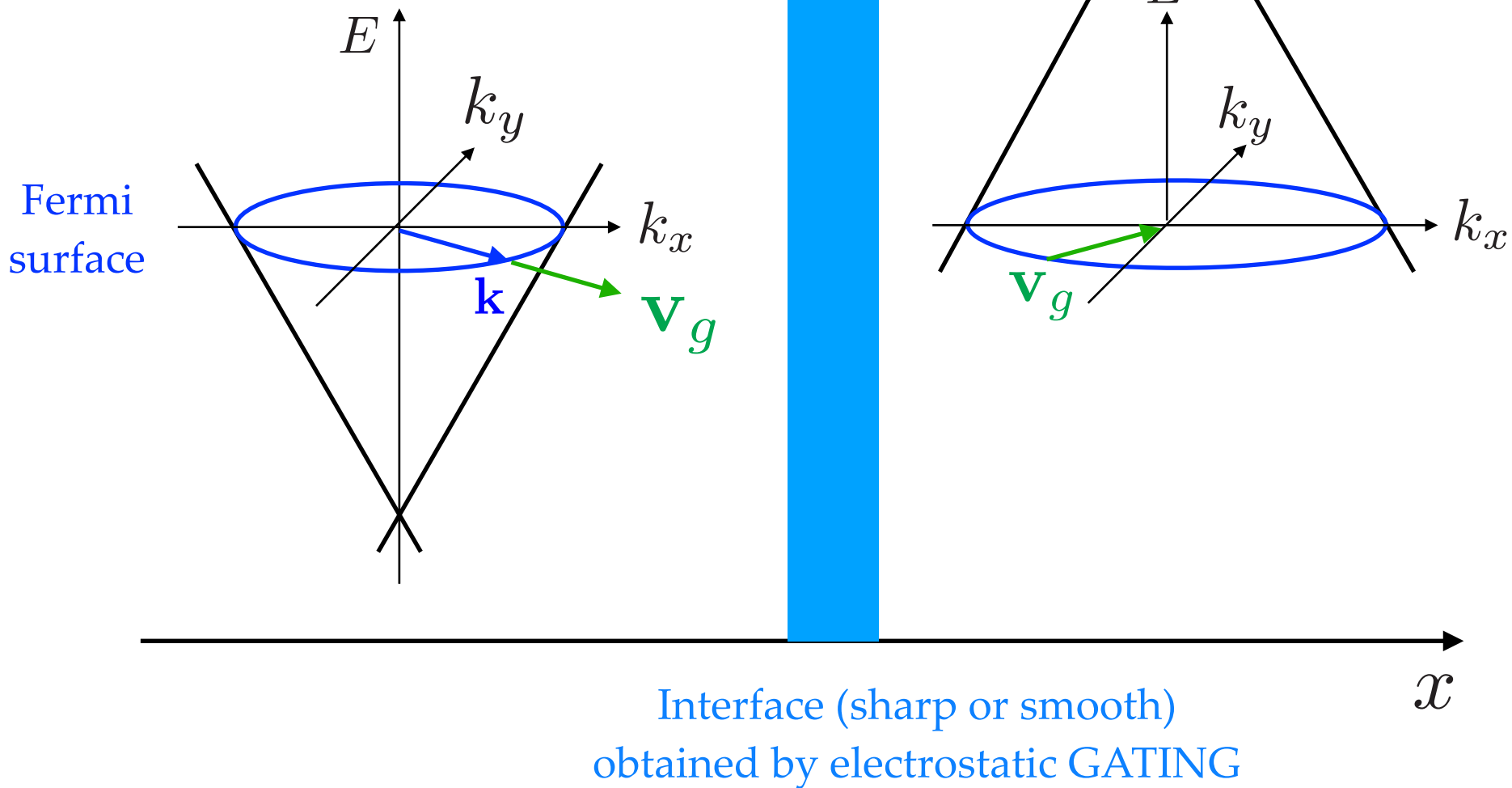
In n-doped graphene, the group velocity and the wave-vector like points **in the same direction.**

p-doped graphene

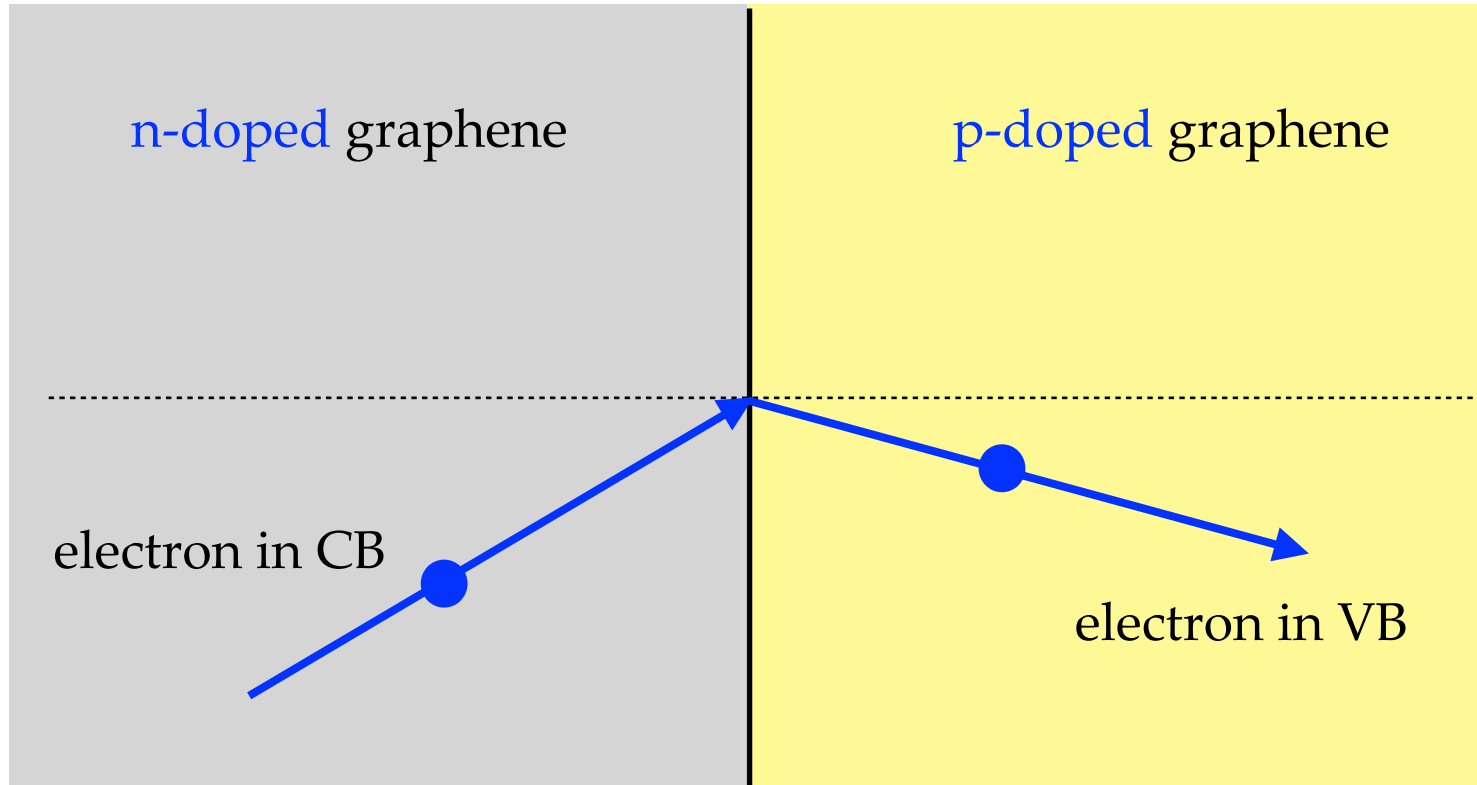


In p-doped graphene, the group velocity and the wave-vector like points in **opposite directions**, like in a **negative index medium**
(The same in any negatively-dispersing band)

pn junction



Veselago lensing with electrons in graphene

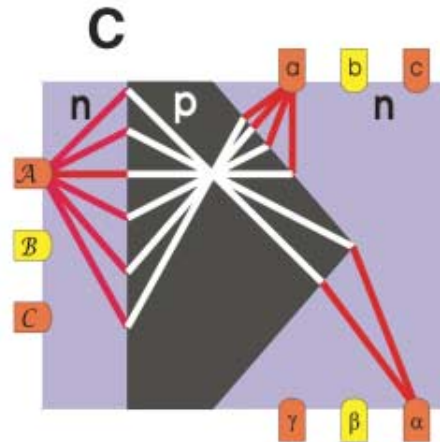
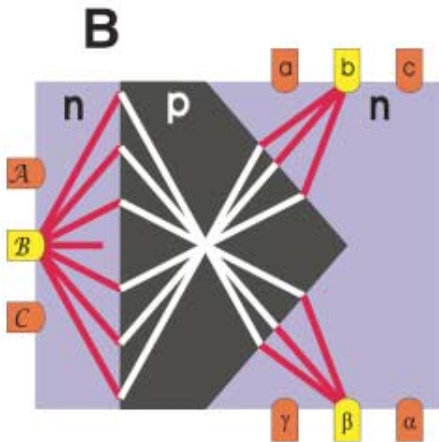
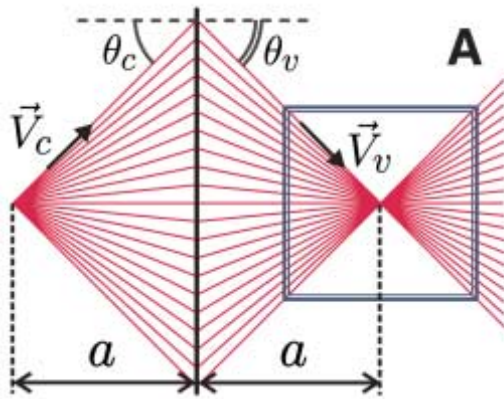
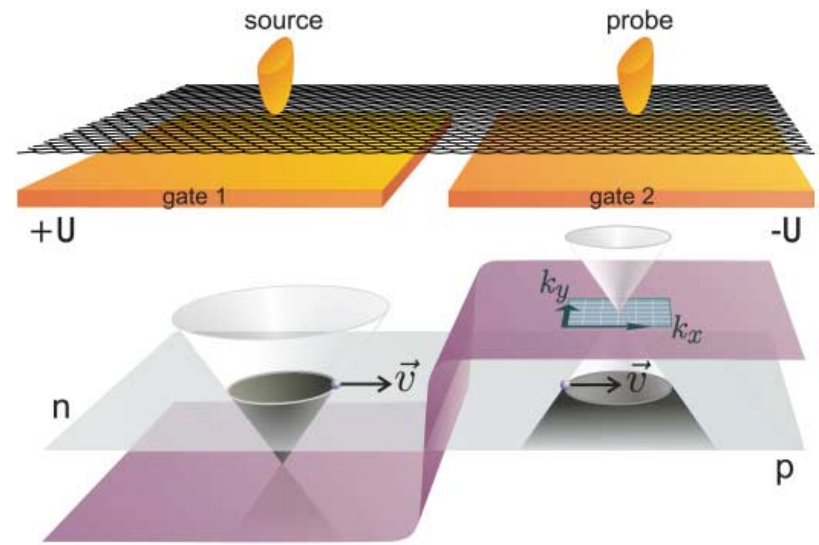


Arrows represent group velocities / semiclassical trajectories

The Focusing of Electron Flow and a Veselago Lens in Graphene p - n Junctions

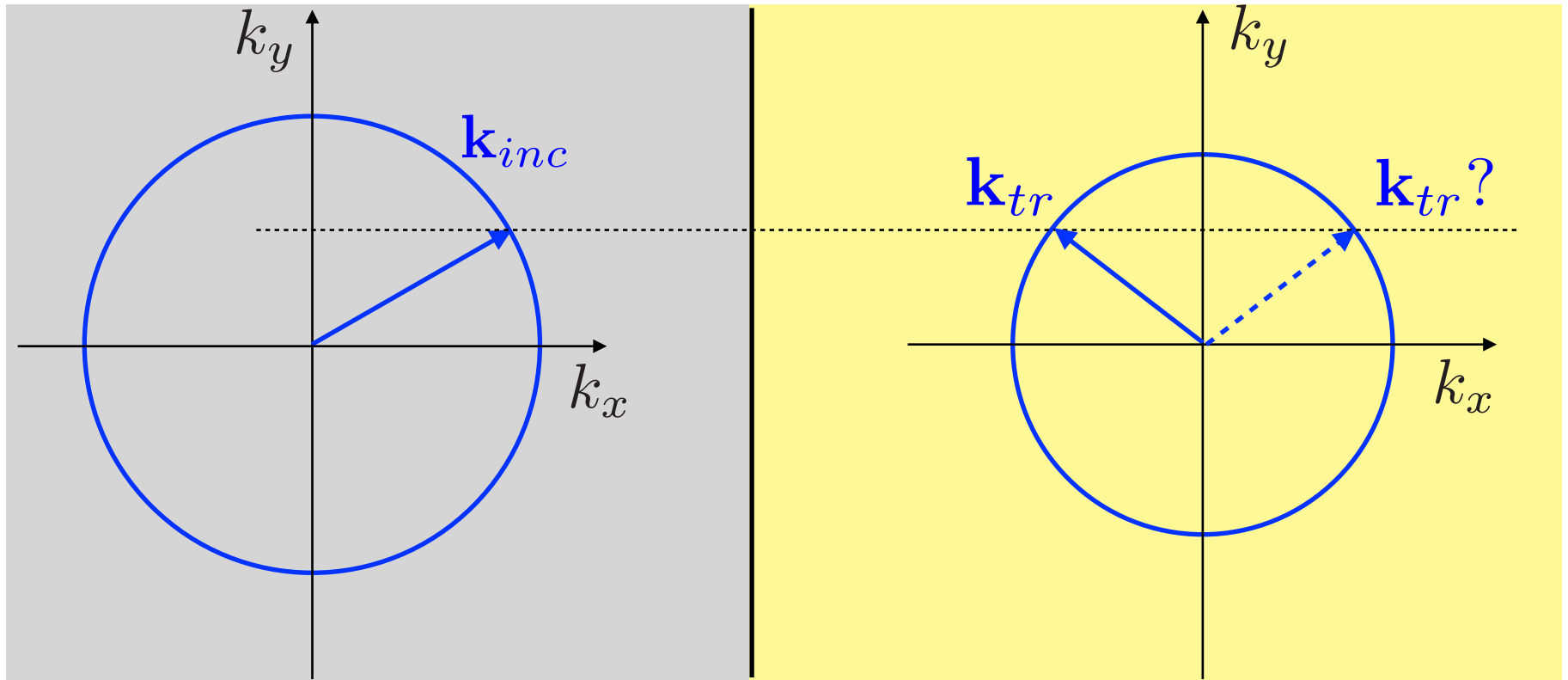
Vadim V. Cheianov,^{1*} Vladimir Fal'ko,¹ B. L. Altshuler^{2,3}

2 MARCH 2007 VOL 315 SCIENCE



Experimental realization
 Quentin Wilmart (LPENS, ex-LPA)

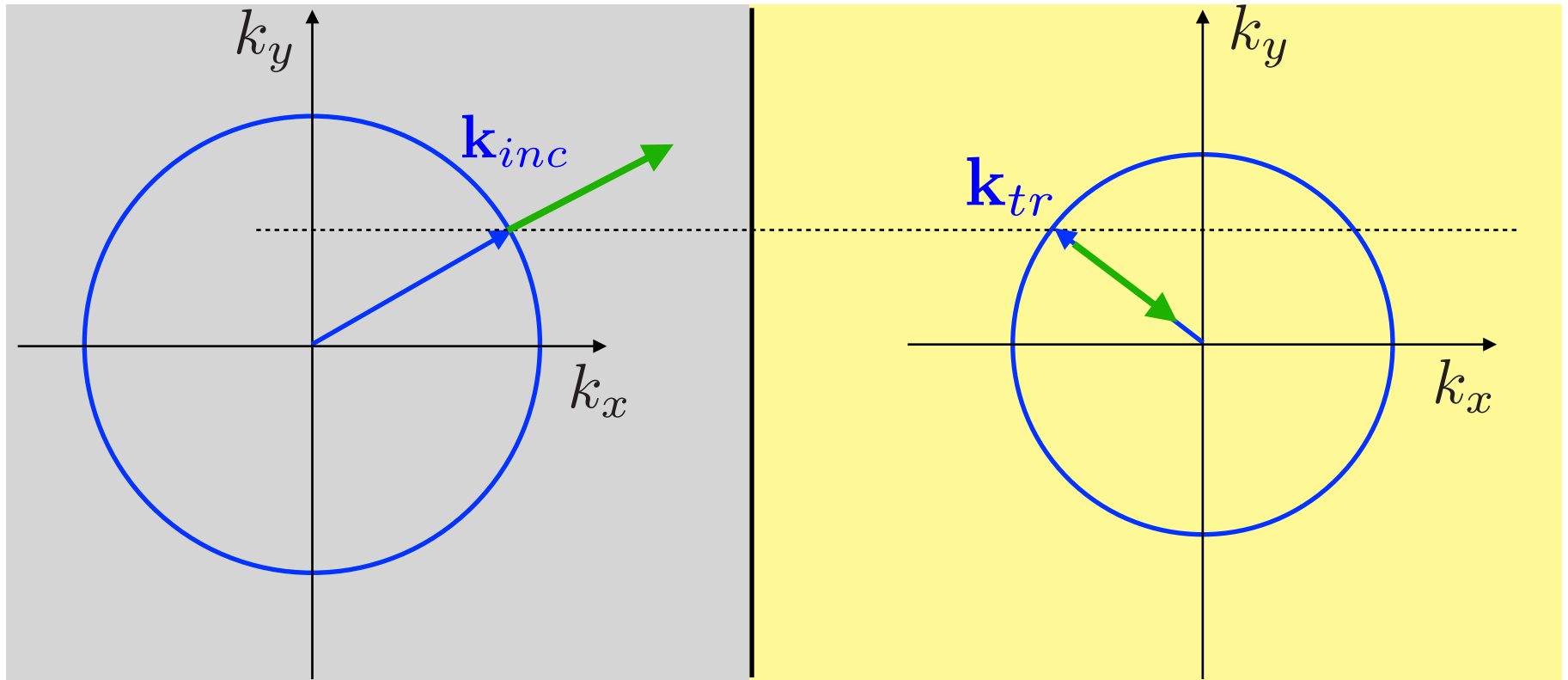
Origin of negative refraction



n-doped graphene

p-doped graphene

Origin of negative refraction



n-doped graphene

p-doped graphene

Transmission through pn junction

Perfect transmission at **normal incidence**

Sharp interface : $k_F d \ll 1$

$$\lambda_F \simeq 10 - 100 \text{nm}$$

$$T(\phi) = \cos^2 \phi$$

Smooth interface : $k_F d \gg 1$

$$T(\phi) = e^{-\pi k_F d \sin^2 \phi}$$

Collimation

Experiments in ballistic regime (2015)

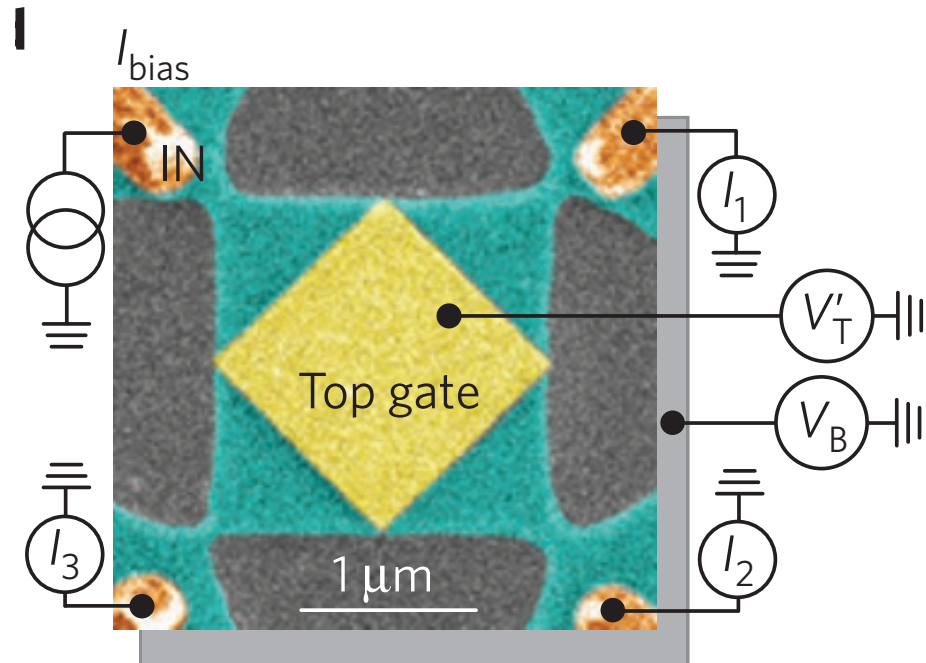
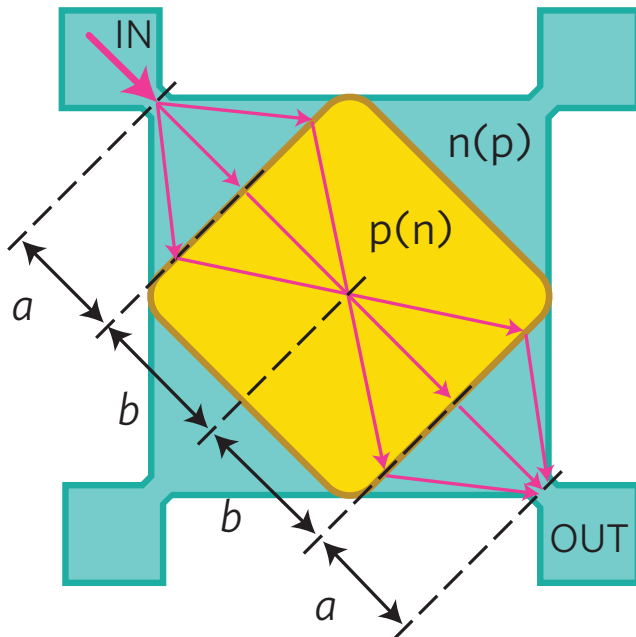
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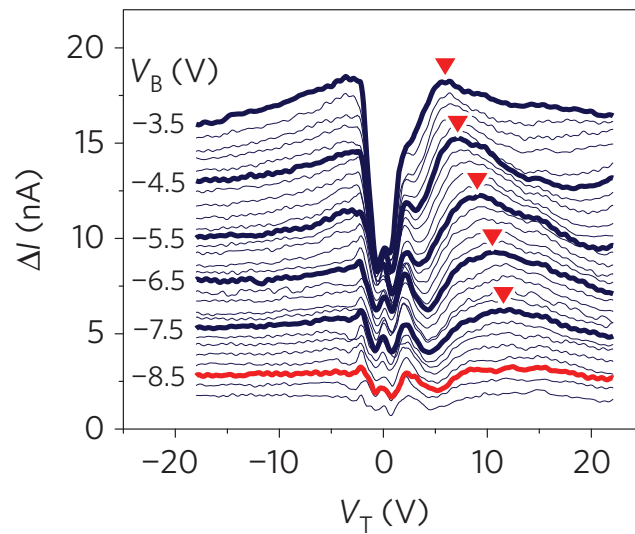
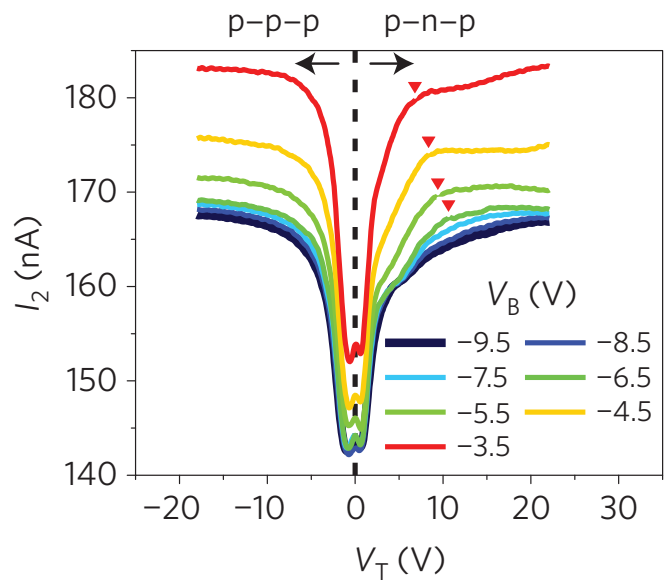
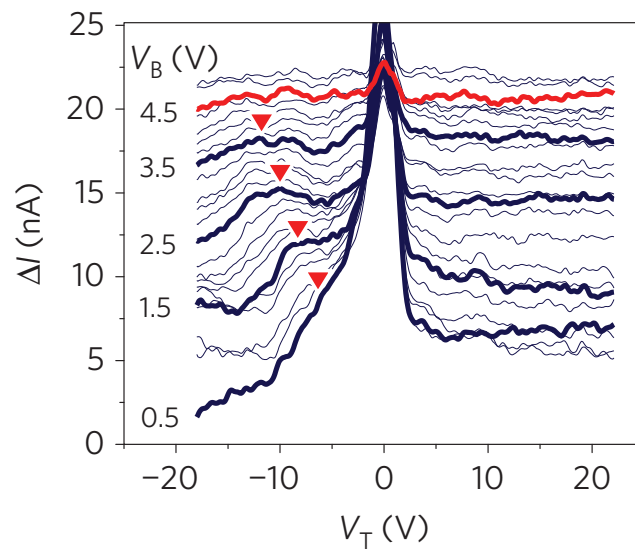
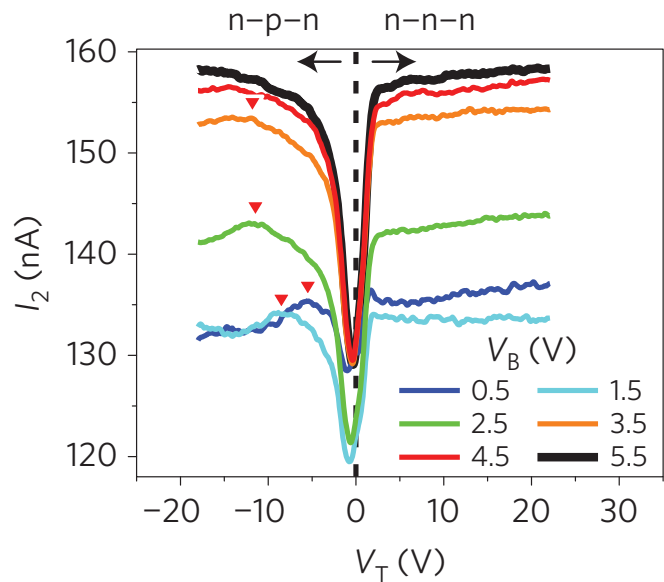
PUBLISHED ONLINE: 14 SEPTEMBER 2015 | DOI: 10.1038/NPHYS3460

Observation of negative refraction of Dirac fermions in graphene

Gil-Ho Lee[†], Geon-Hyoung Park and Hu-Jong Lee^{*}



Focused peaks...



Experiments in ballistic regime (2016)

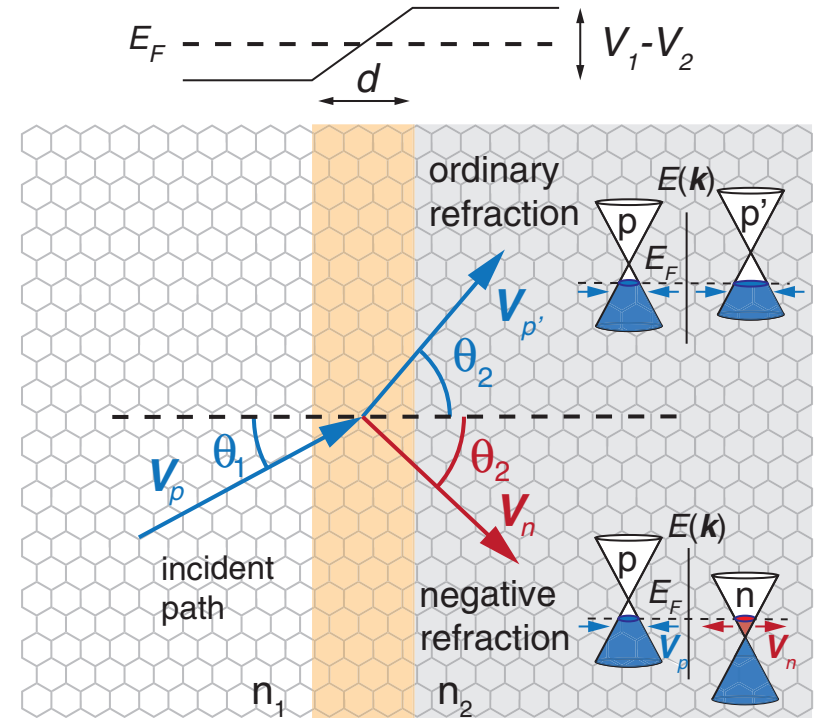
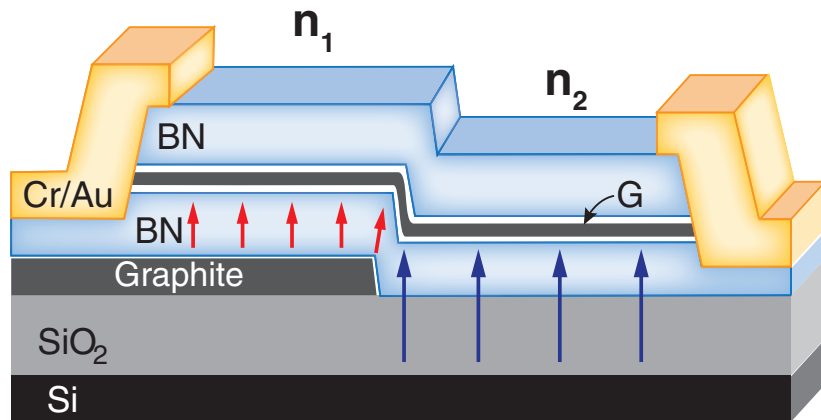
RESEARCH | REPORTS

GRAPHENE

Electron optics with p-n junctions in ballistic graphene

Shaowen Chen,^{1,2*} Zheng Han,^{1,7*} Mirza M. Elahi,³ K. M. Masum Habib,^{3,†} Lei Wang,⁴ Bo Wen,^{1,8} Yuanda Gao,⁵ Takashi Taniguchi,⁶ Kenji Watanabe,⁶ James Hone,⁵ Avik W. Ghosh,³ Cory R. Dean^{1,‡}

1522 30 SEPTEMBER 2016 • VOL 353 ISSUE 6307



Encapsulated graphene (between BN layers)

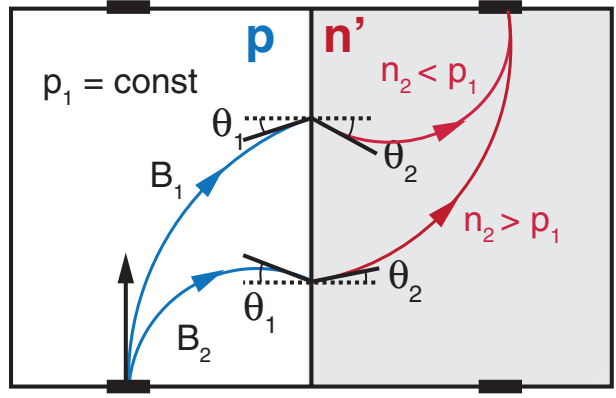
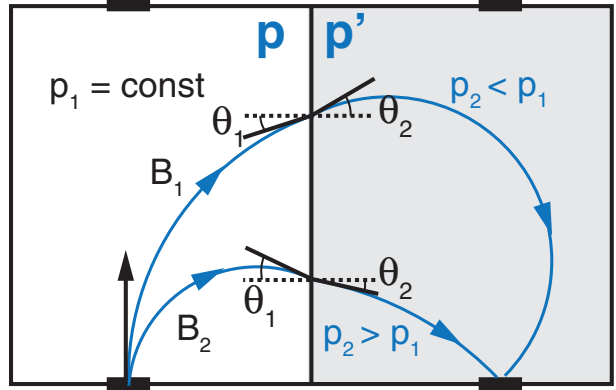
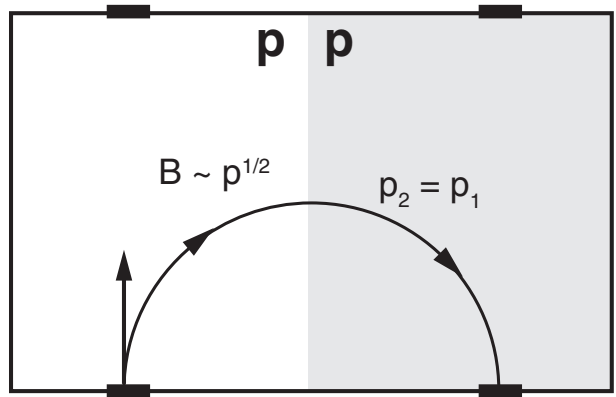
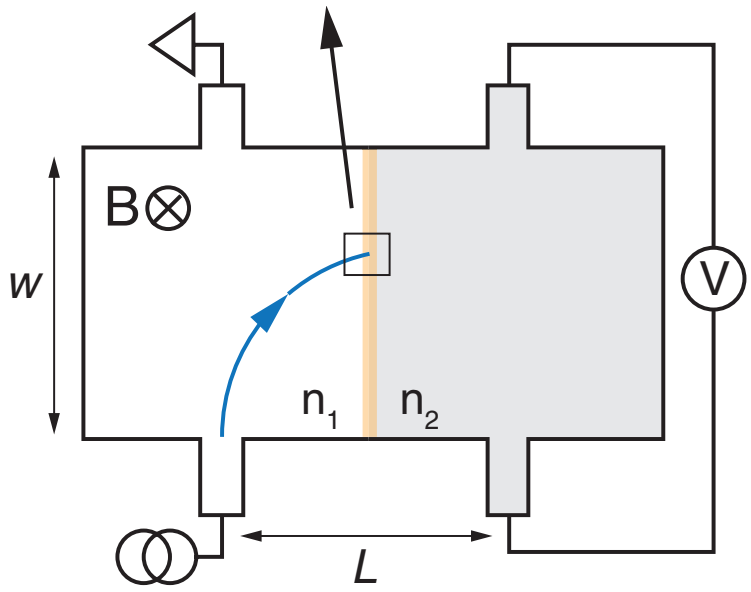
GRAPHENE

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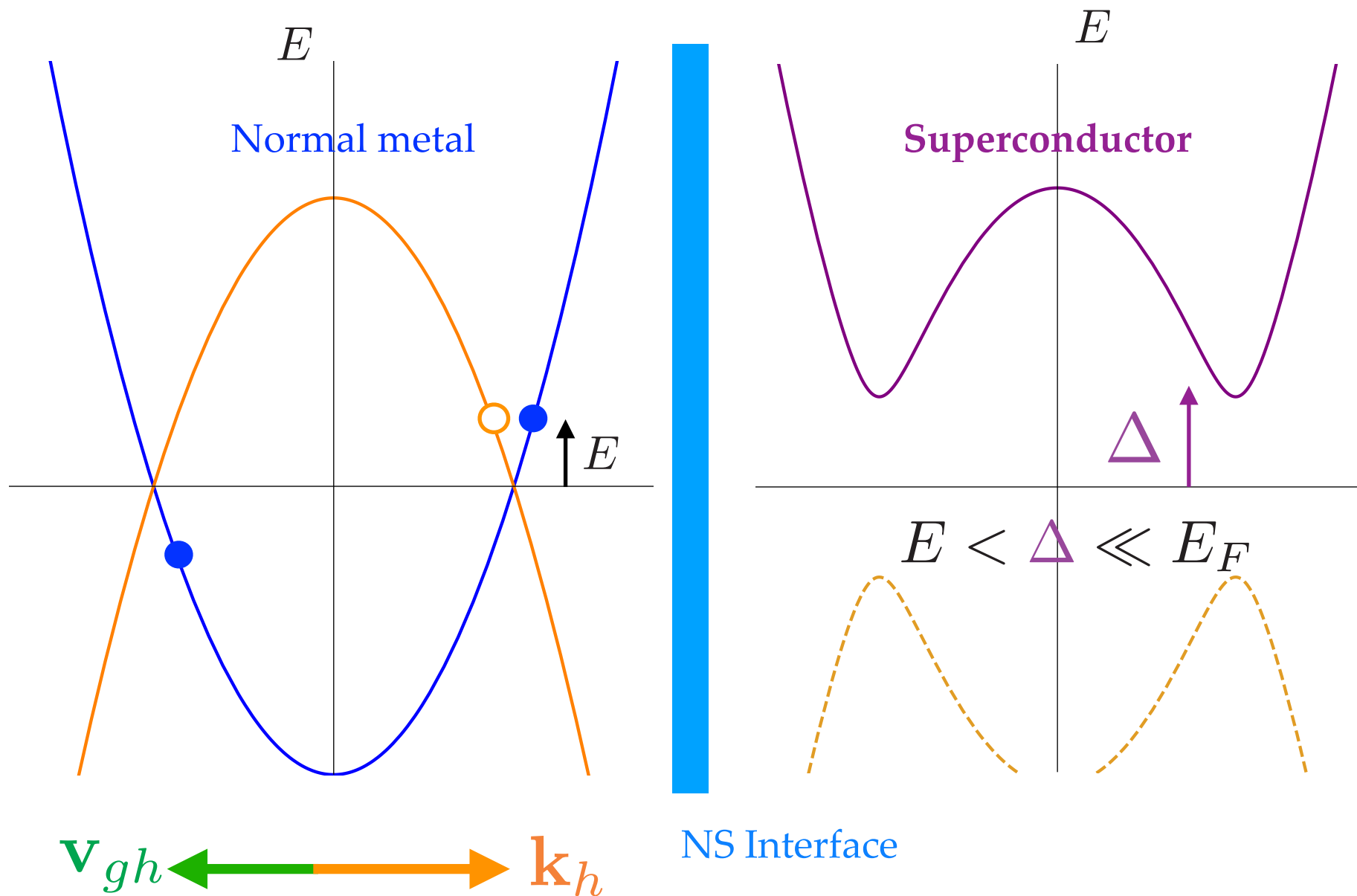
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Negative refraction and cyclotron focusing

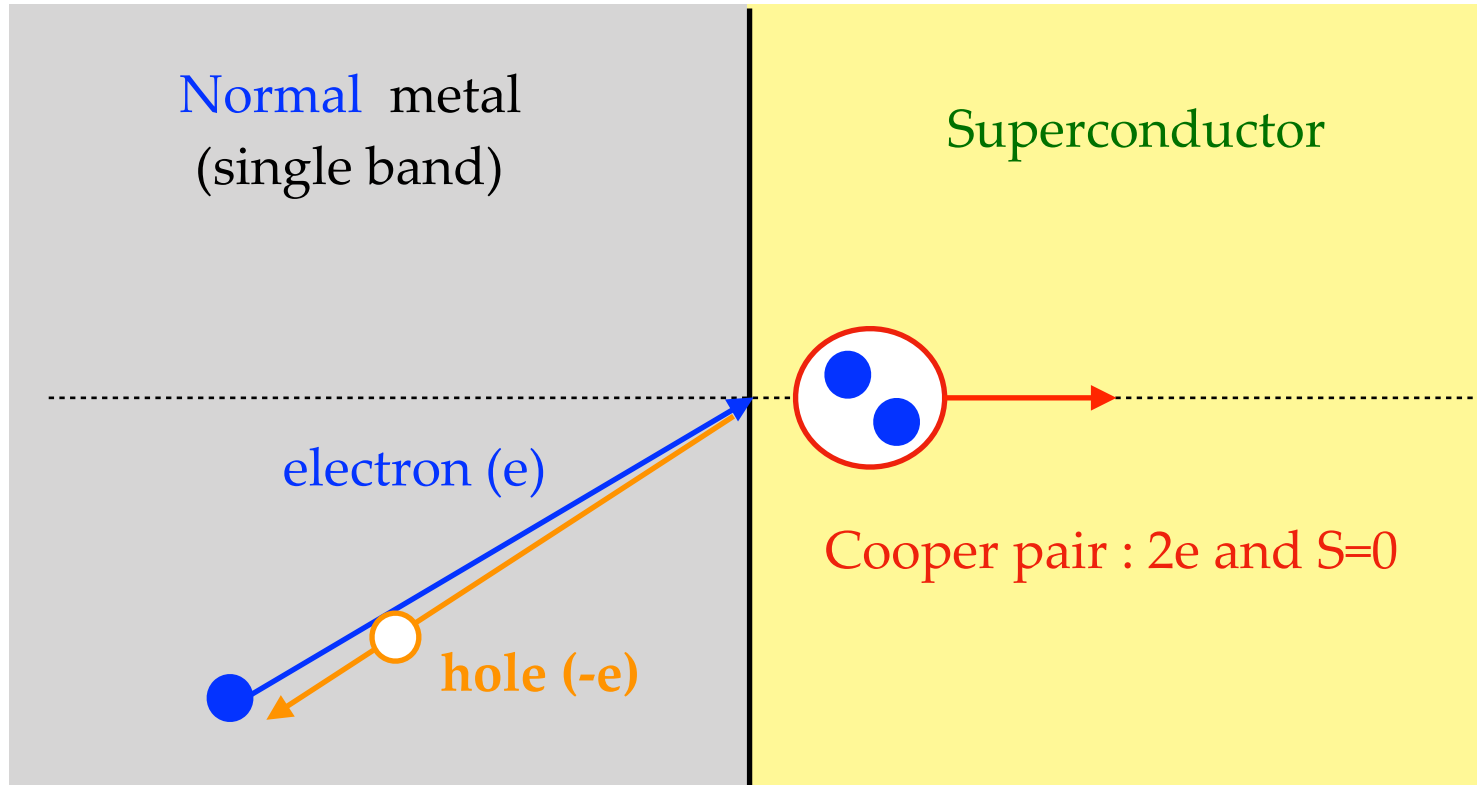


III) Andreev reflection at graphene/ superconductor interface

Andreev reflection at NS interface

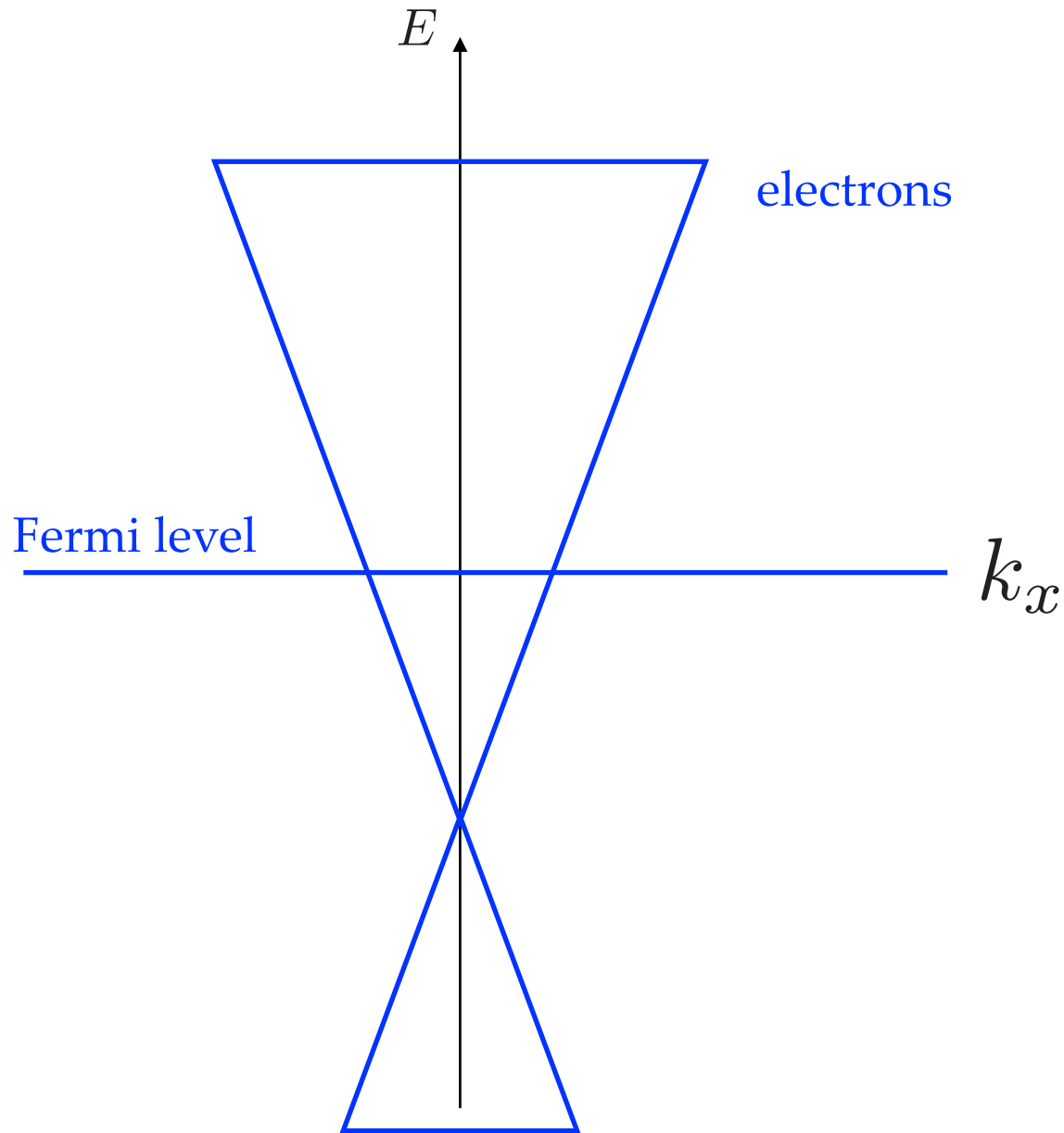


Andreev reflection in standard metals

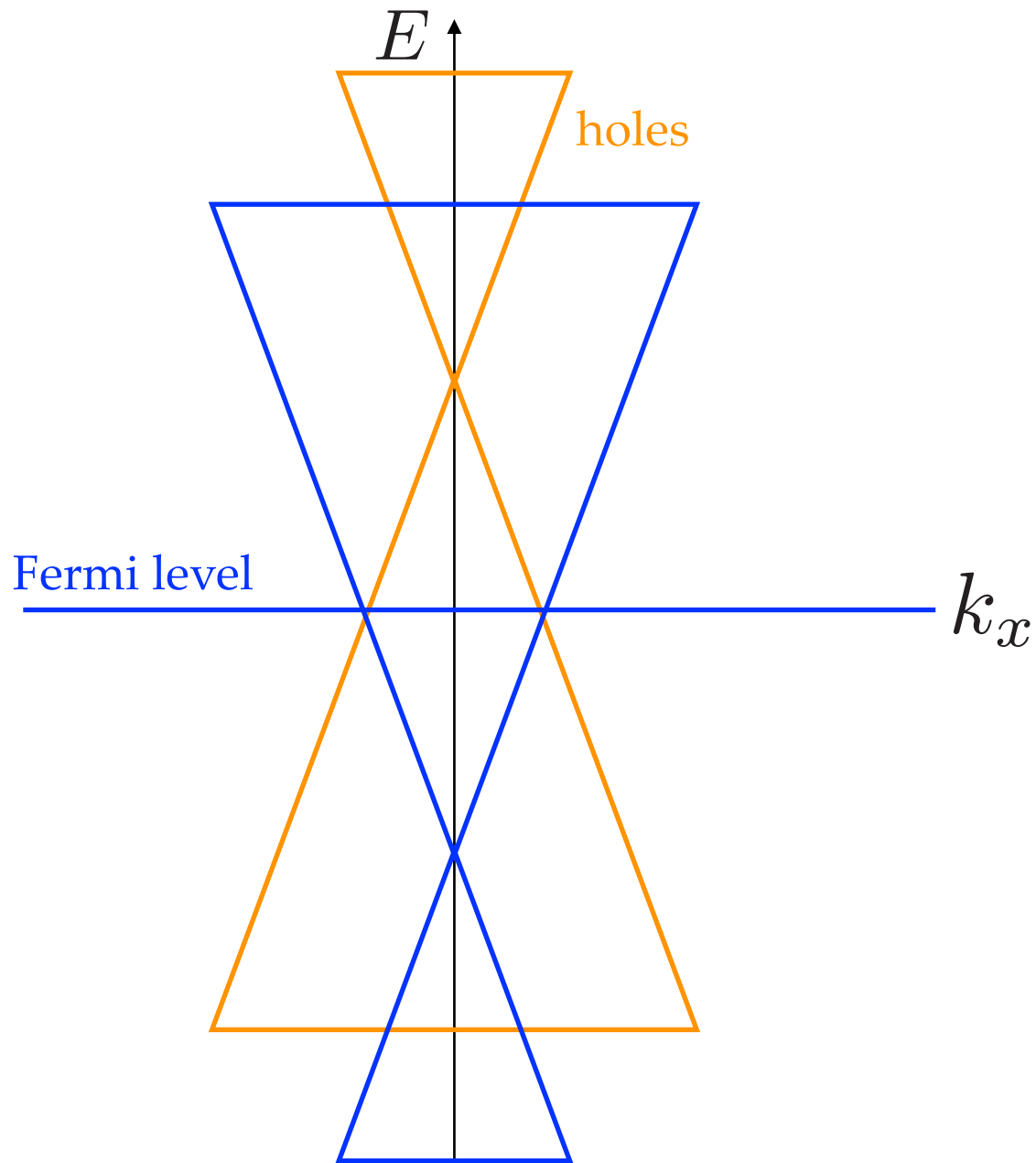


Retro-reflection (with a tiny mismatch angle)

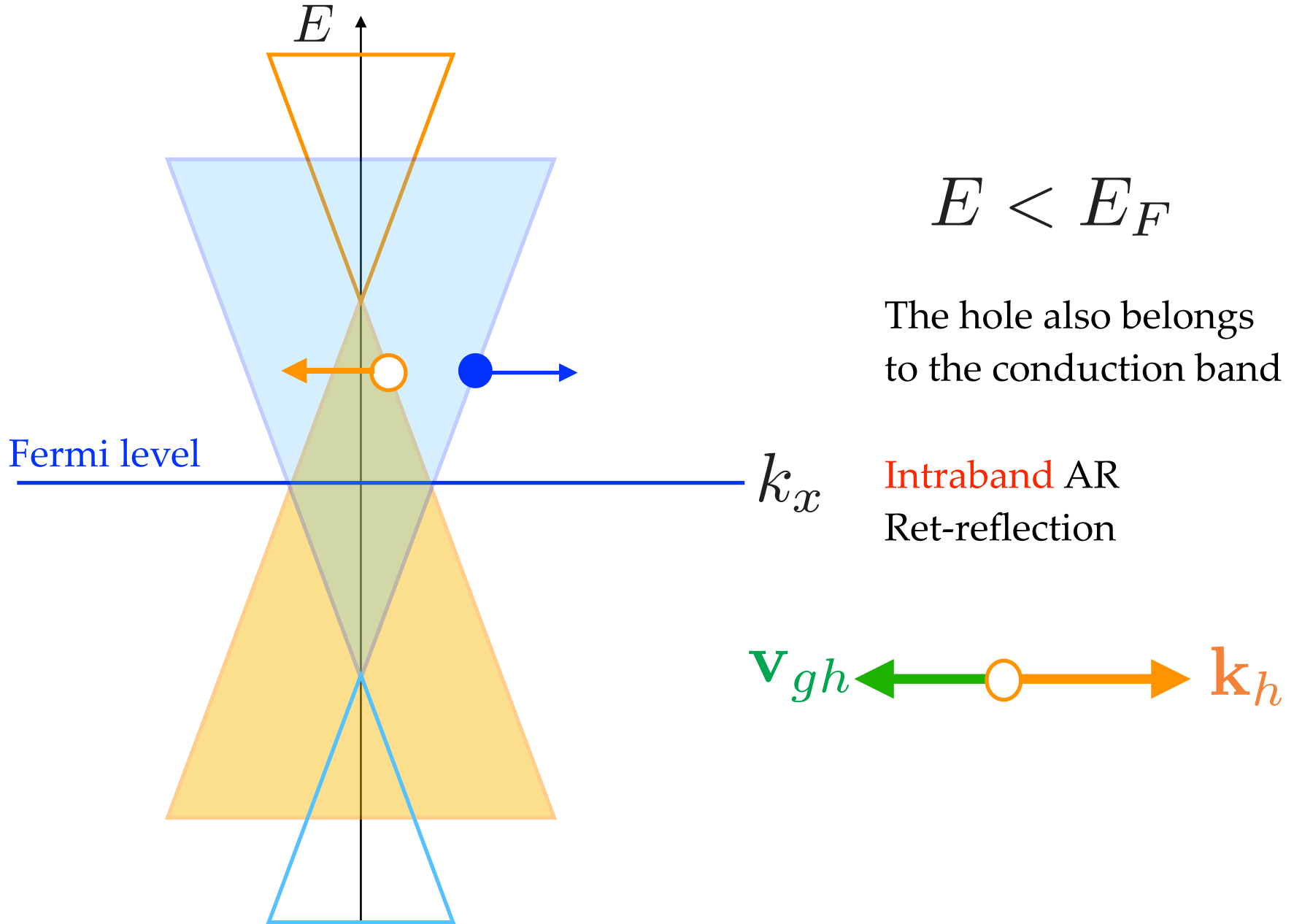
Andreev reflection in a Dirac cone (2 bands)



Andreev reflection in a Dirac cone

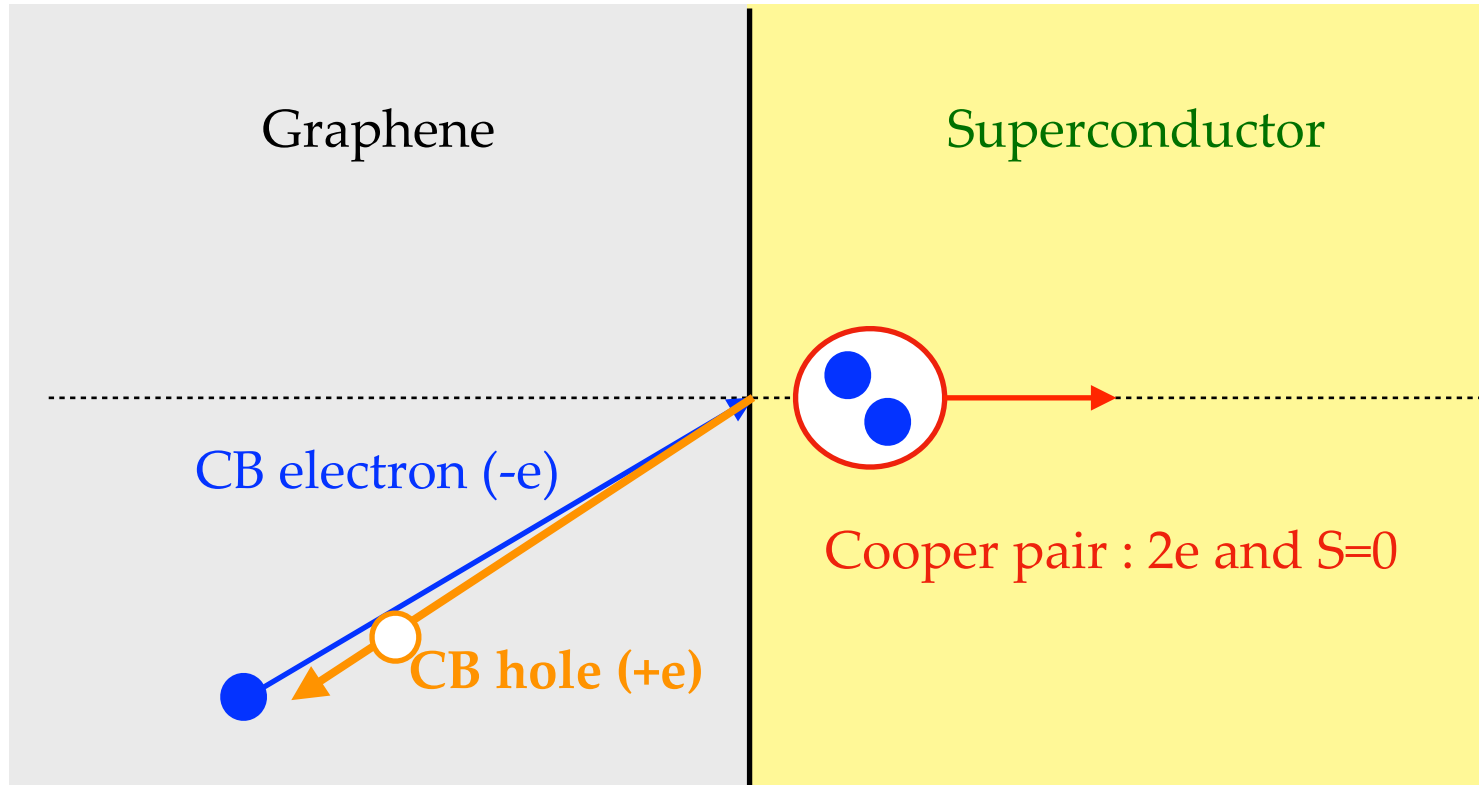


Andreev-Dirac reflection : intraband case



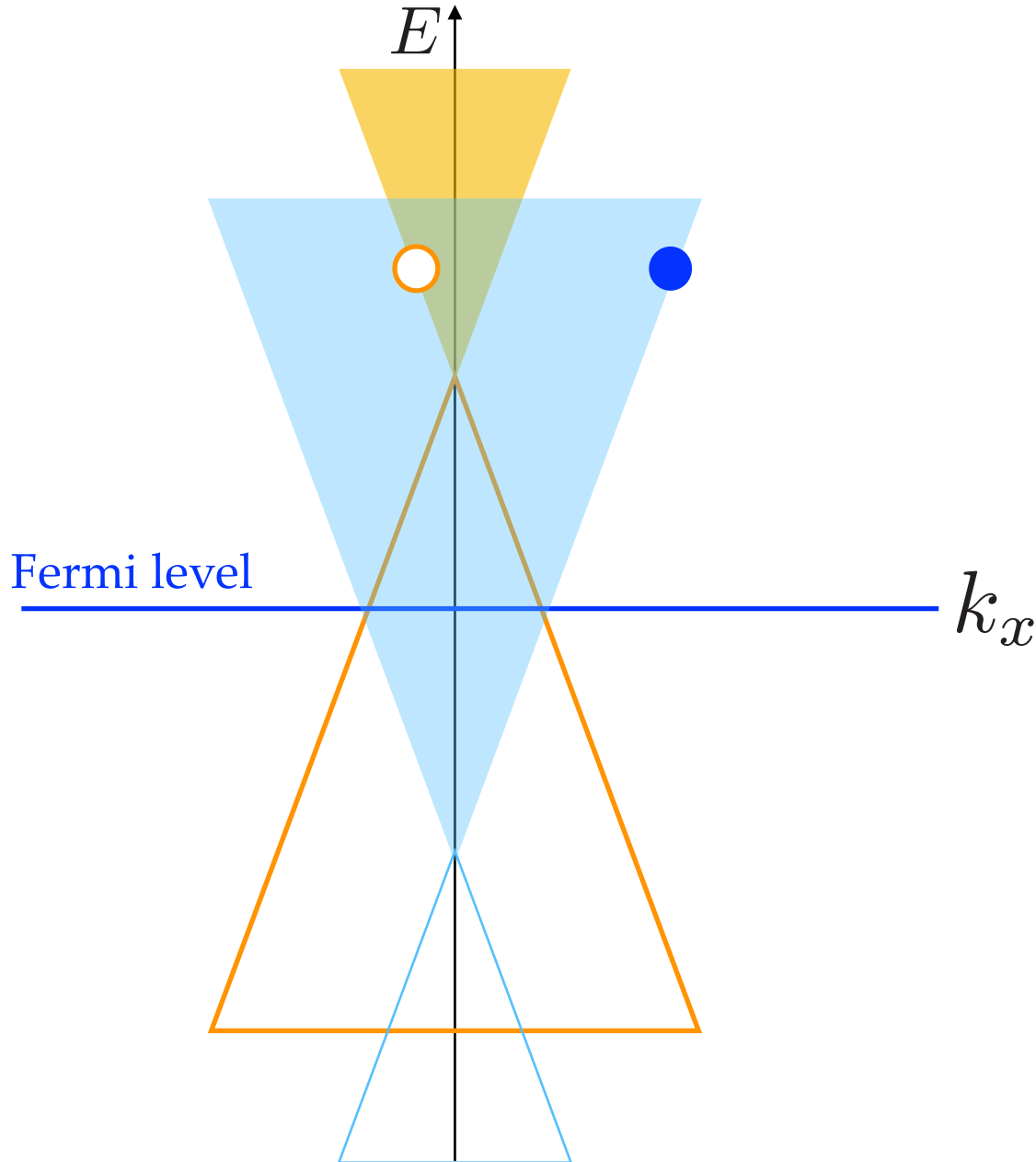
Andreev-Dirac reflection : intraband case

Intraband



Retro-reflection (with a tiny mismatch angle)

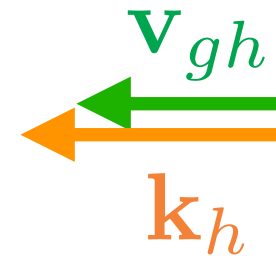
Andreev-Dirac reflection : interband case



$$E > E_F$$

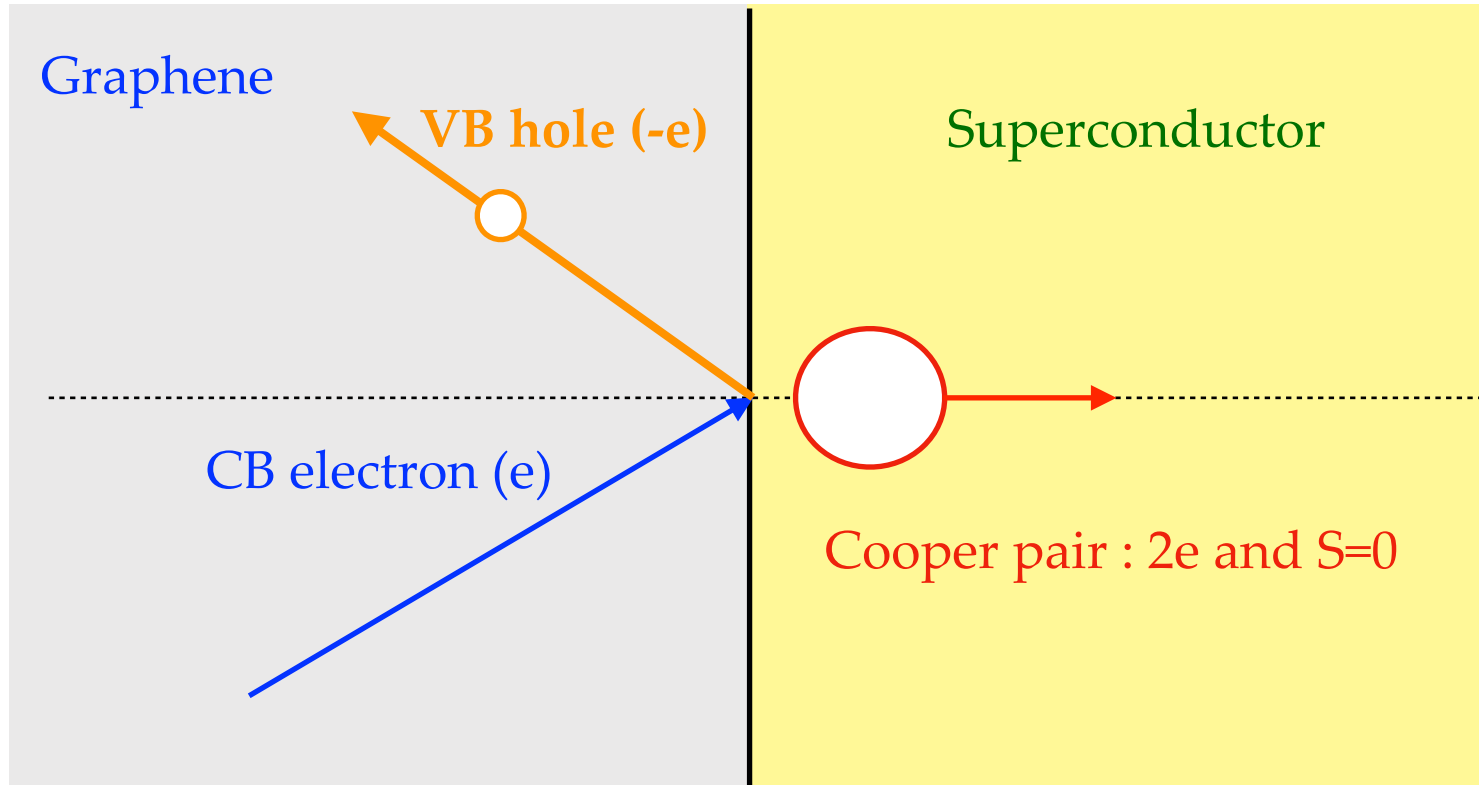
The hole also belongs to the valence band

Interband AR
Retro-reflection



Andreev-Dirac reflection : interband case

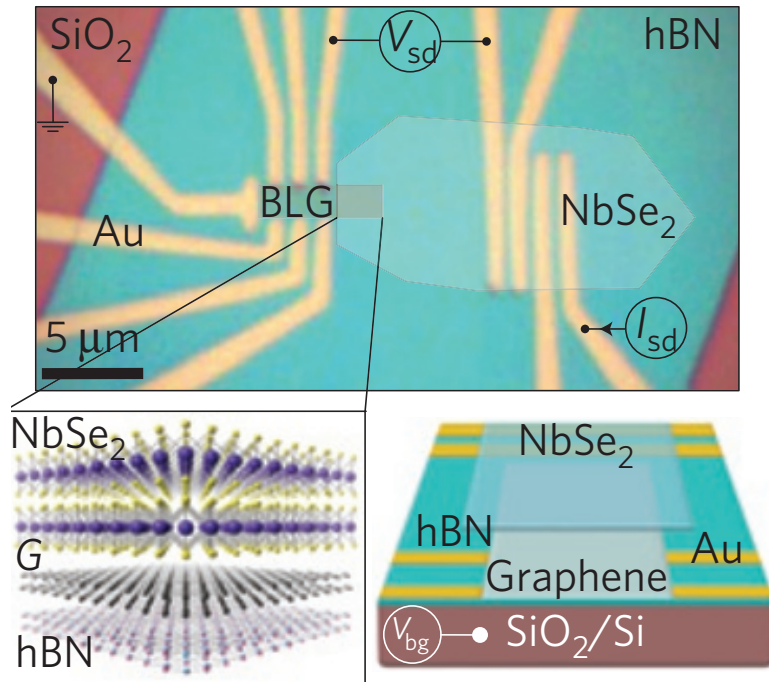
Interband



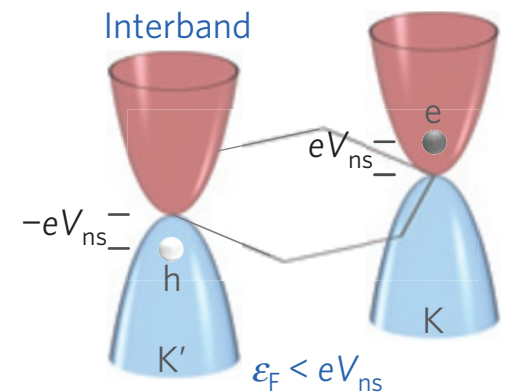
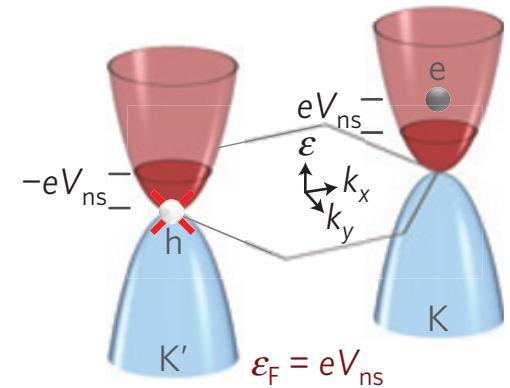
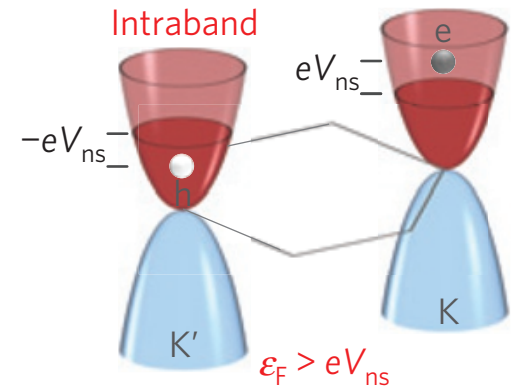
Specular-reflection (with a tiny mismatch angle)

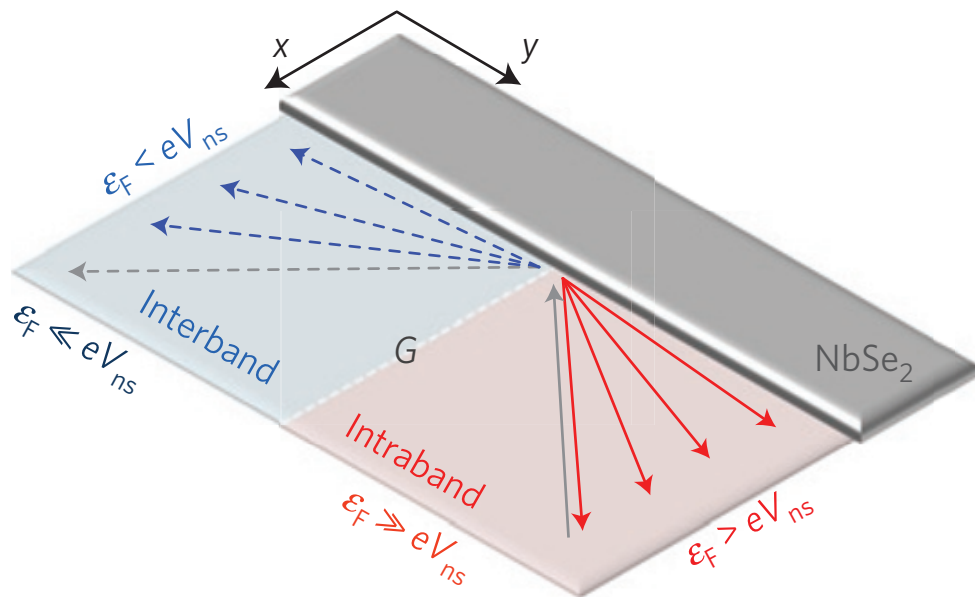
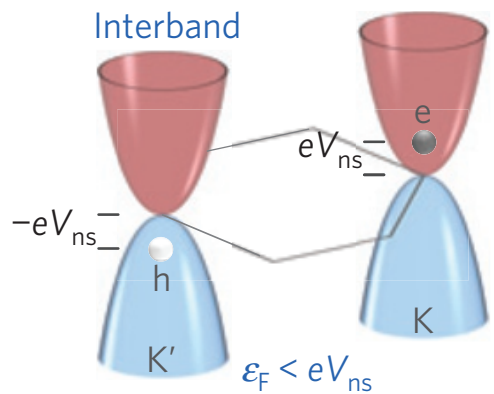
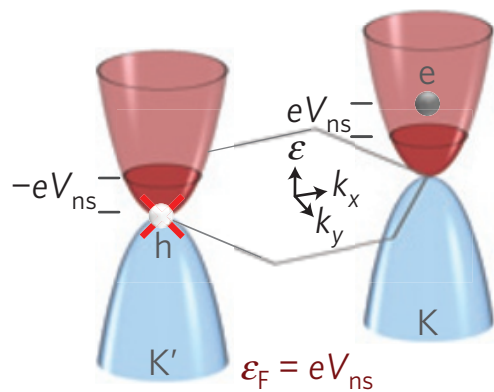
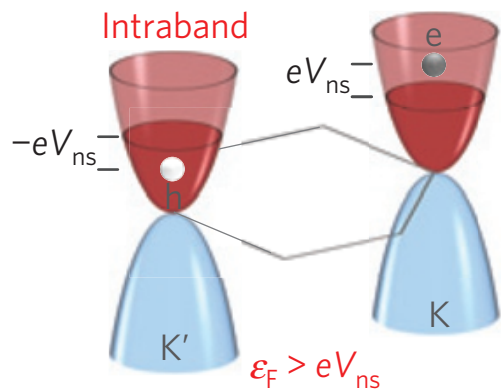
Specular interband Andreev reflections at van der Waals interfaces between graphene and NbSe₂

D. K. Efetov^{1*}, L. Wang², C. Handschin¹, K. B. Efetov^{3,4}, J. Shuang⁵, R. Cava⁵, T. Taniguchi⁶,
K. Watanabe⁶, J. Hone², C. R. Dean¹ and P. Kim^{1*}



Bilayer Graphene





Internal degrees of freedom

Veselago lensing is useful because it allows focusing by a planar interface, but at this stage it is **spin/chirality insensitive**

In devices using electronic optics, it would be interesting to design lenses that **focus a specific eigenvalue** of spin or chirality.



Spintronics



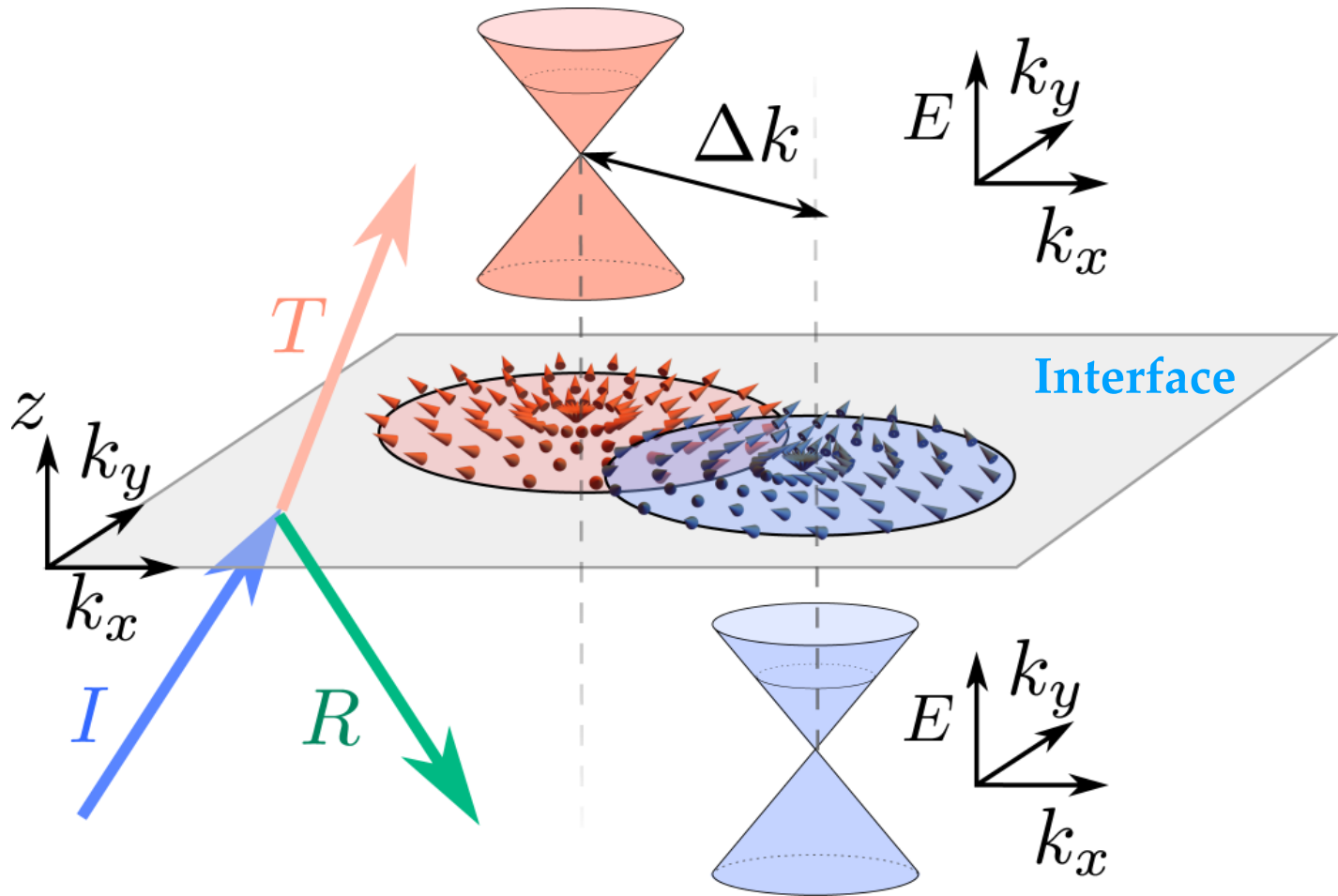
Valleytronics

Chirality?

IV) Veselago lensing in 3D Weyl semi-metals

S. Tchoumakov, J. Cayssol, and A.G. Grushin, PRB 105, 075309 (2022)

Chirality filtering



First idea : shifting the Weyl nodes in momentum space

3D Veselago lensing

Recipe : Replace graphene by a 3D Weyl/Dirac semimetal

3 Issues :

- 1) Large carrier density but **hard to tune it by gating**
- 2) Bad interfaces because 2 different materials (disorder)
- 3) Insensitivity to internal degree of freedom : spin or chirality

Solution :

Use **the chiral anomaly** to create selectively a pn junction for one chirality only

Chirality of a Weyl fermion

Chirality : projection of spin along momentum

$$H_R = v\sigma \cdot (\mathbf{p} - \mathbf{K})$$

Positive chirality

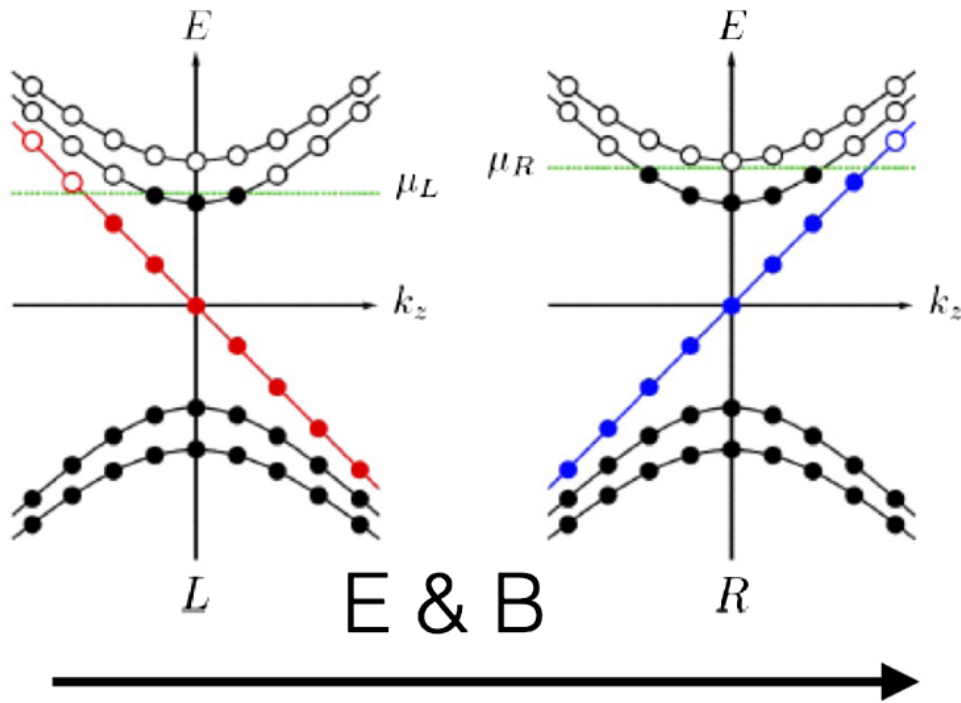
$$H_L = -v\sigma \cdot (\mathbf{p} + \mathbf{K})$$

Negative chirality

Total chirality is zero : even number of Weyl nodes (Nielsen-Ninomiya)

Chiral anomaly

Simplest case : two Weyl nodes of opposite chiralities



Landau levels (3D)

$$\frac{dn_{R/L}^{3D}}{dt} = \pm \frac{e^2}{h^2} \mathbf{E} \cdot \mathbf{B}$$

Nielsen-Ninomiya (1983)

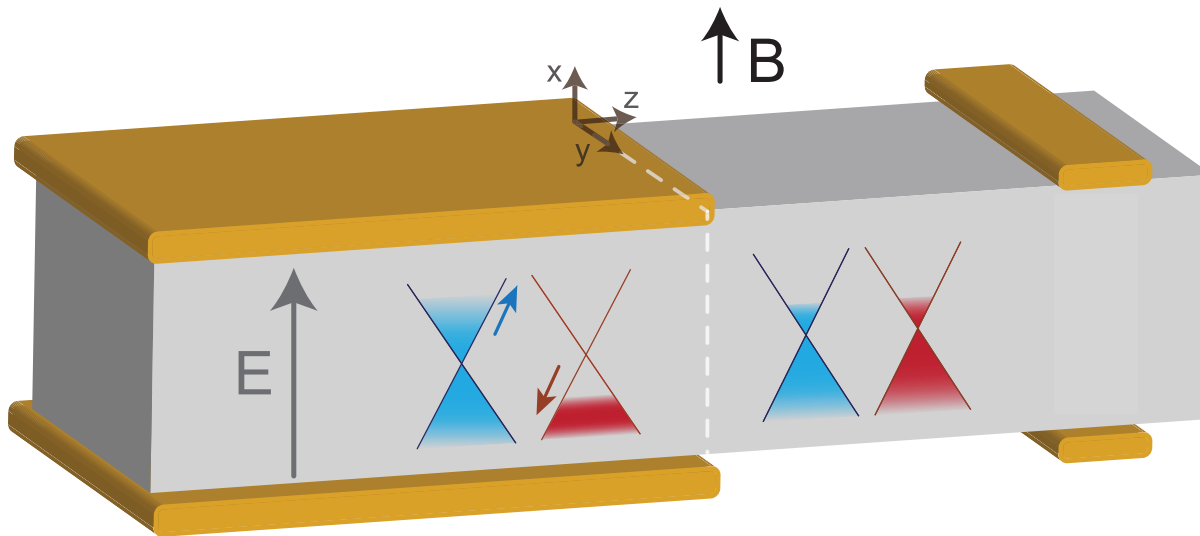
Effect of **B** : disperse along the B field direction only

Effect of **E** : push/pump electrons from one node to the other

Interface and chiral anomaly

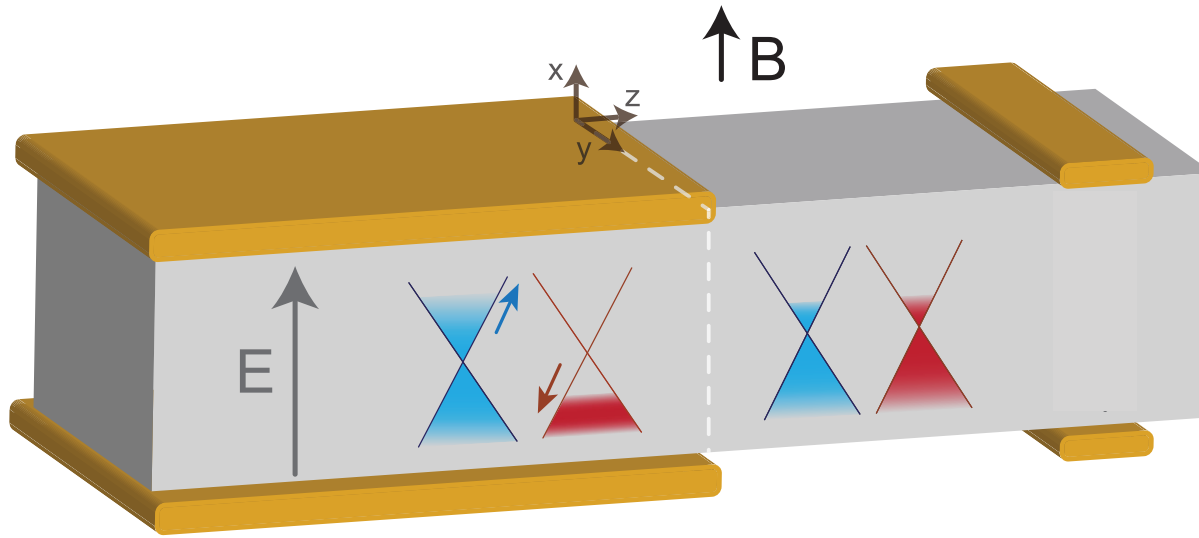
$$\Delta n = \frac{\tau e^2}{2\pi^2 \hbar^2} \mathbf{E} \cdot \mathbf{B}$$

Interface : two Weyl nodes of opposite chiralities
Magnetic field is applied homogeneously
Same material (clean interface)



pn junction when the magnetic field exceeds a critical value
(of order 1 Tesla)

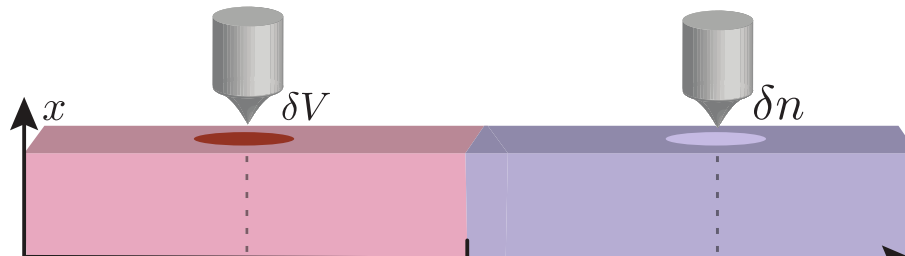
Interface and chiral anomaly



pn junction : the red chirality is Veselago focused by the interface

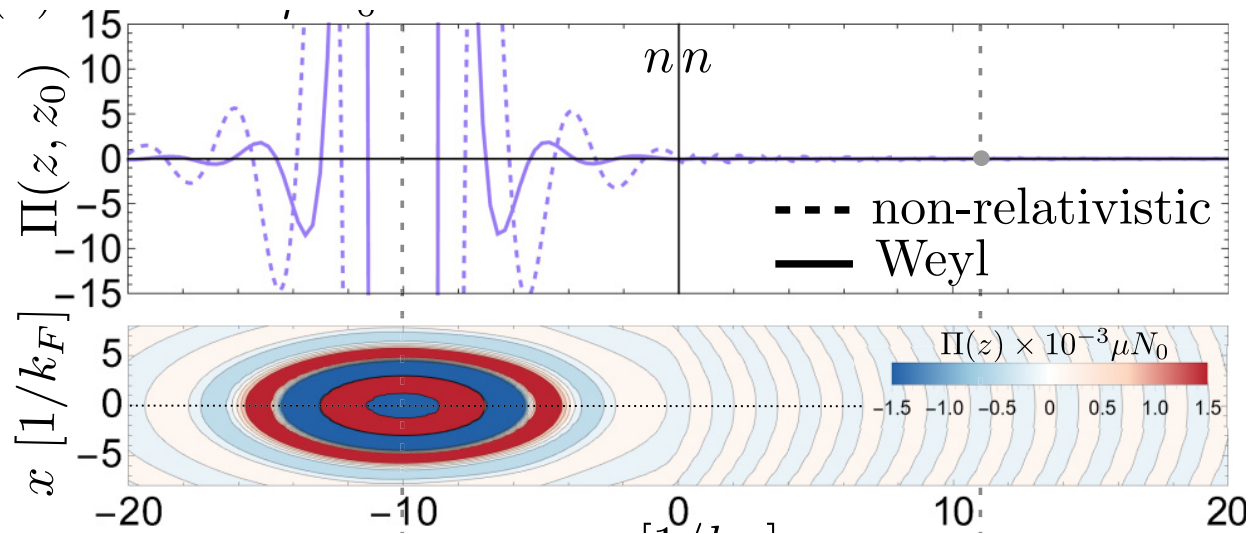
nn' junction : the blue chirality is not

Polarizability and STM experiments



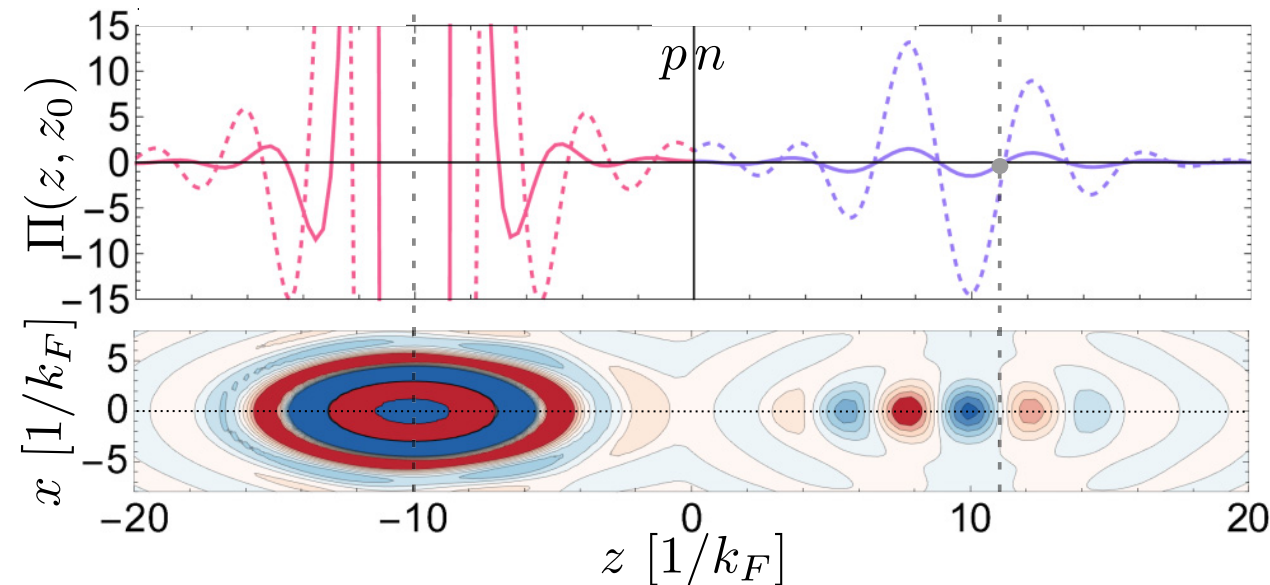
$$\Pi(z, z') = -\frac{1}{2\pi} \int d\omega \text{Tr}[\hat{G}(z, z')\hat{G}(z', z)]$$

Polarizability results



nn junction

Damped Friedel
oscillations



np junction

Formation of a
(charge) image

Conclusion

Veselago effect allows focalisation of a beam by a single flat interface between materials with « opposite » dispersions. This is implemented by pn junctions in Dirac materials

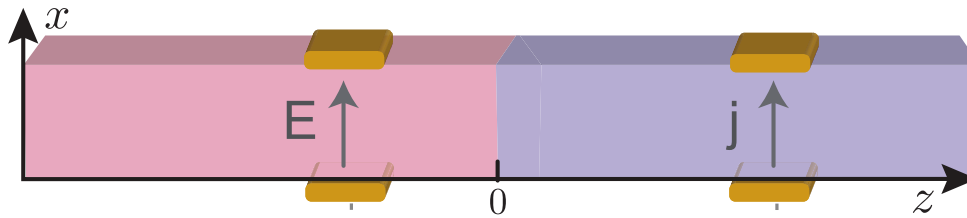
Graphene and Weyl : Zero gap allows transparent interfaces

Those pn junctions can be created either by :

- electrostatic doping (2D)
- the chiral anomaly pumping. In this case, the focusing is also chirality sensitive

Thank you !

Nonlocal transport



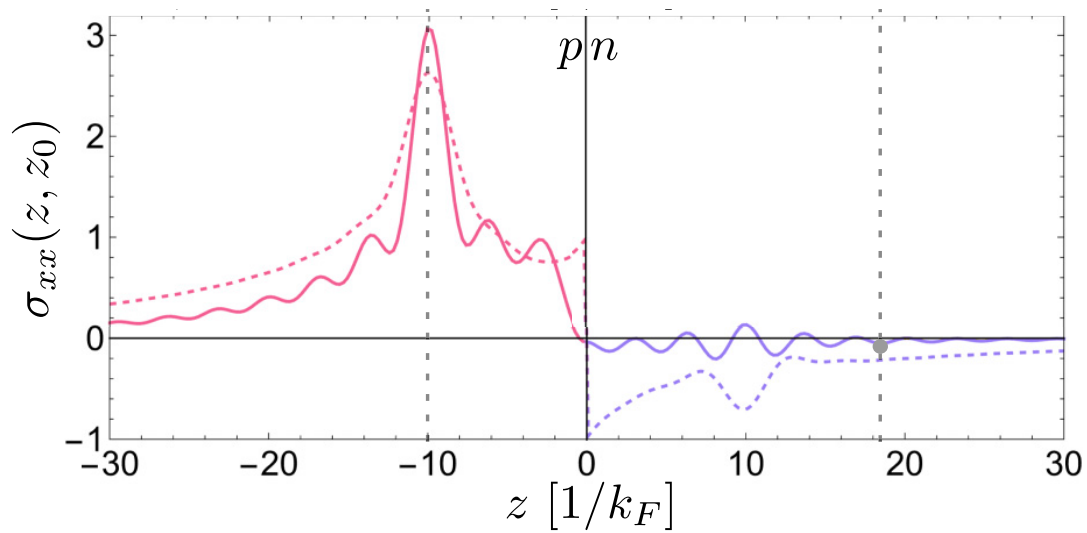
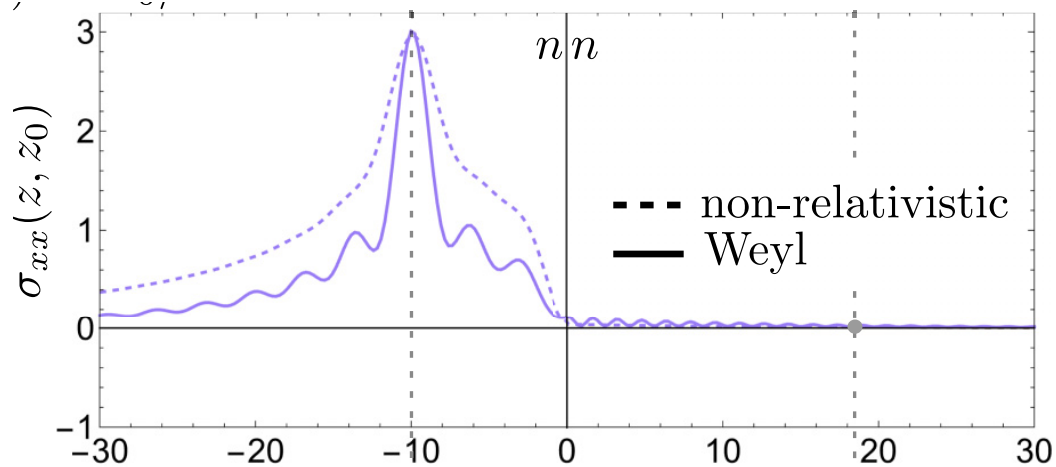
Magnetic field applied homogeneously and beyond a critical field (necessary to generate the pn junction)

Voltage only on one side of the junction

Nonlocal conductivity

$$\sigma_{\mu\nu}(z, z') = \int \frac{dS_z dS_{z'}}{\pi \mathcal{A}} \text{Tr}[\hat{j}_\mu \text{Im} \hat{G}(z, z') \hat{j}_\nu \text{Im} \hat{G}(z, z')]$$

Nonlocal conductivity results



Quantum interference and Klein tunnelling in graphene heterojunctions

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